Fast Quantification of Uncertainty and Robustness with Variational Bayes

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ITT Career Development
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With: Ryan Giordano, Rachael Meager, Jonathan H. Huggins, Michael I. Jordan

- Bayesian inference

- Bayesian inference
  - Complex, modular models

- Bayesian inference
  - Complex, modular models; posterior distribution

- Bayesian inference $p(\theta)$
  - Complex, modular models; posterior distribution

- Bayesian inference $p(x|\theta)p(\theta)$
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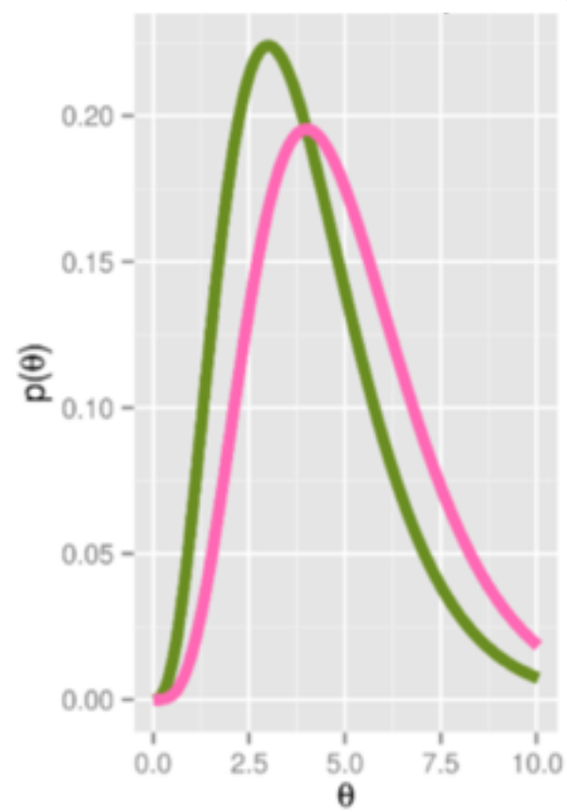
Some reasonable priors



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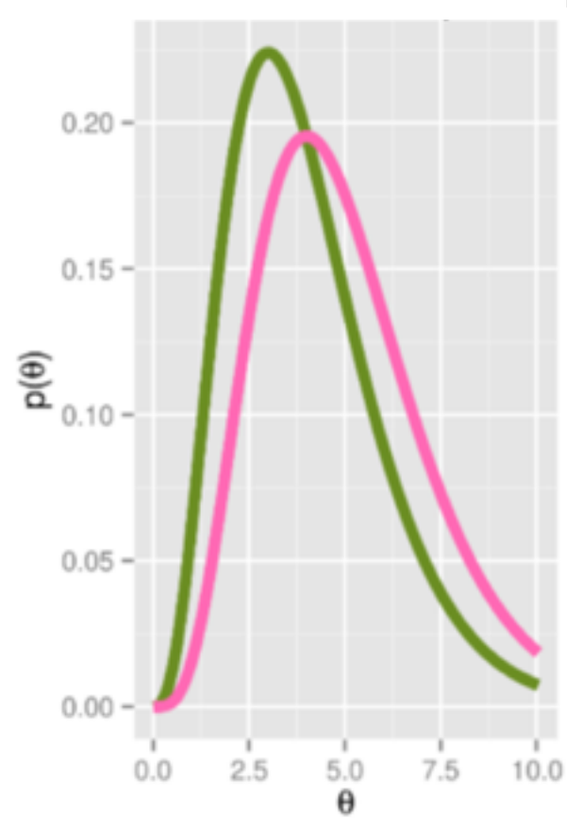
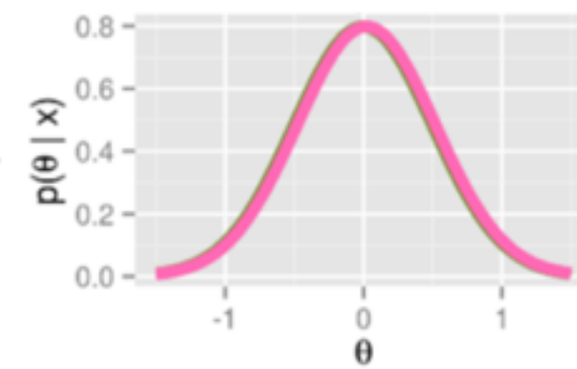


Bayes Theorem



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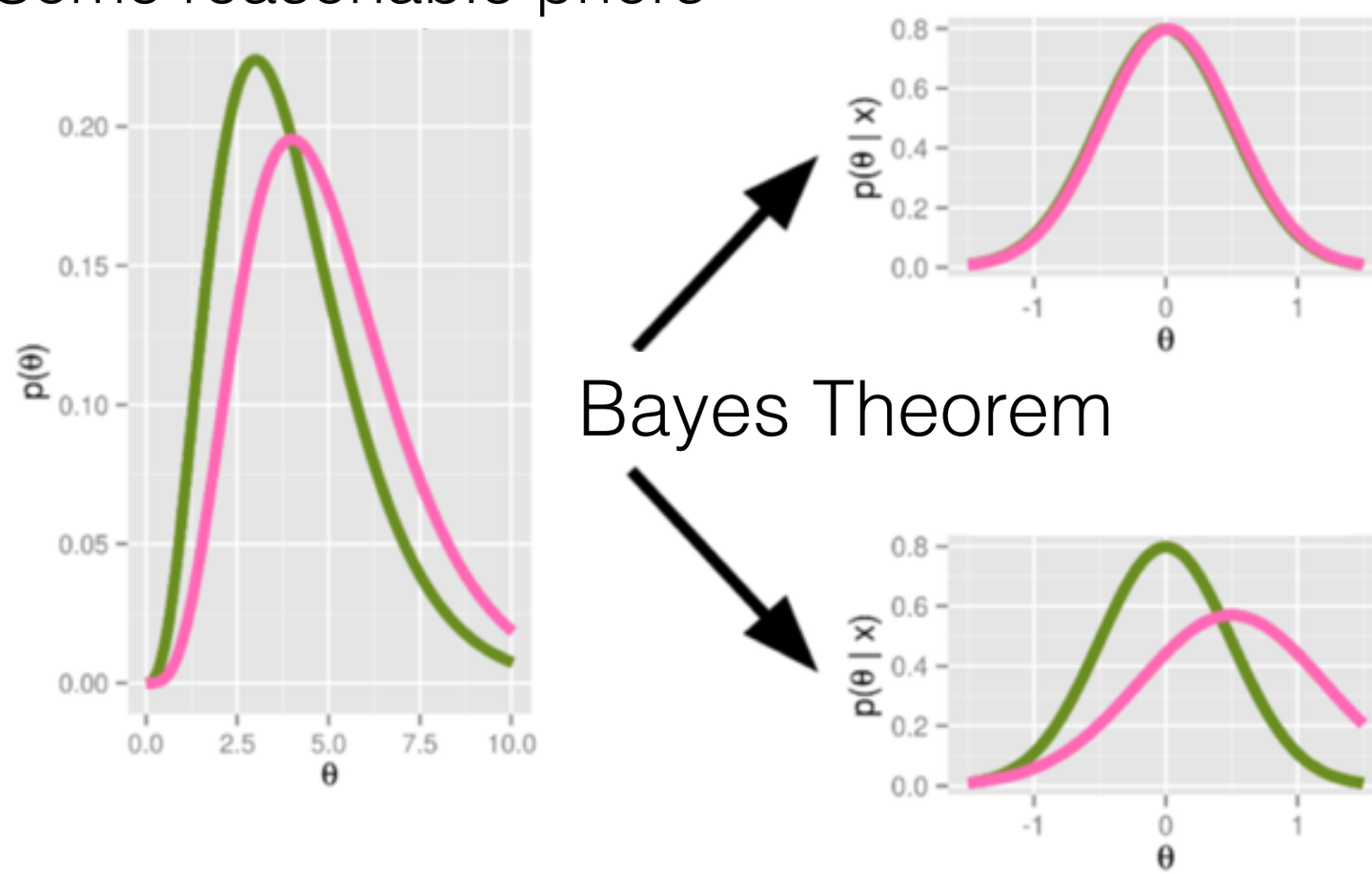

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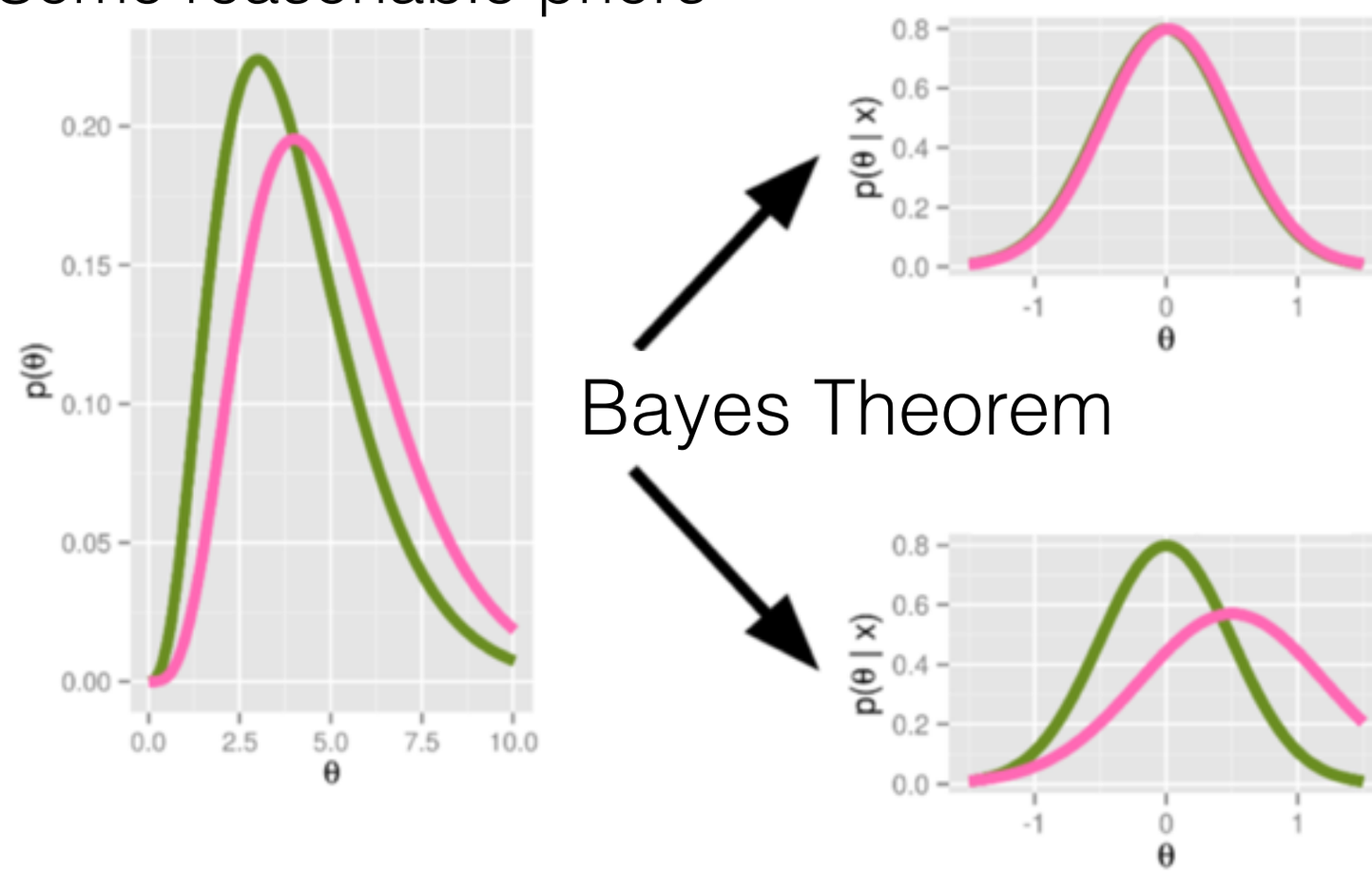

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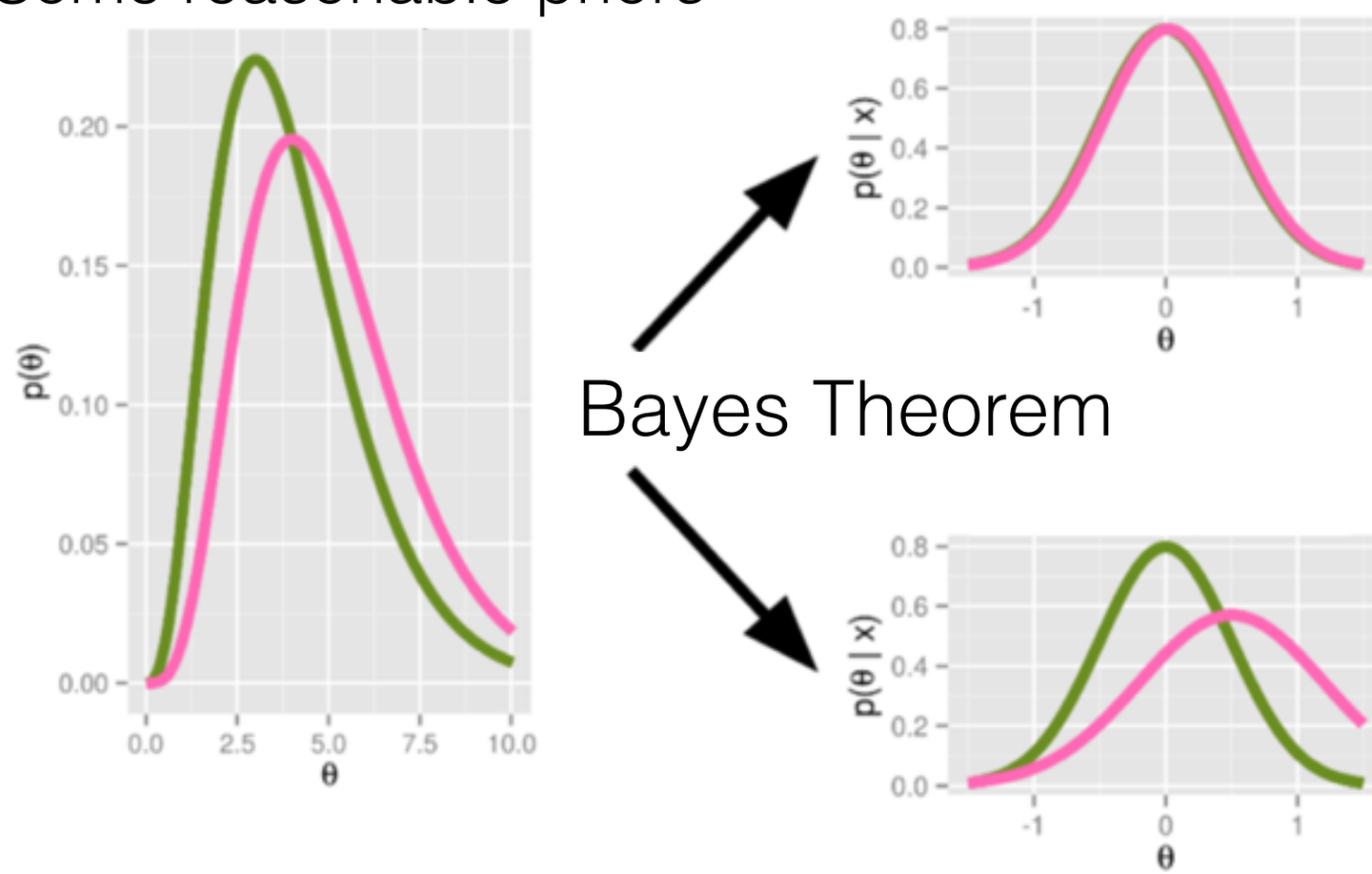

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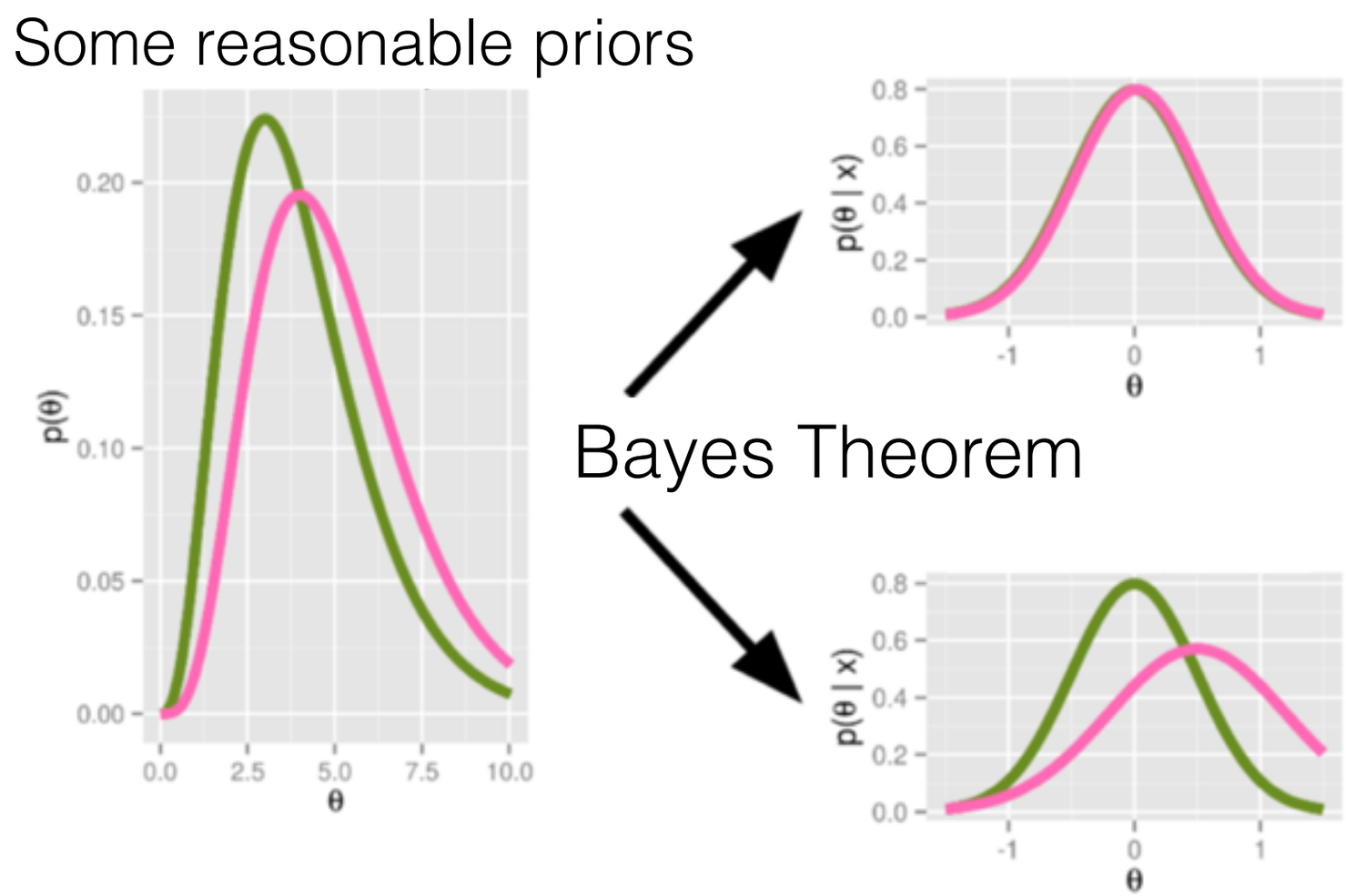

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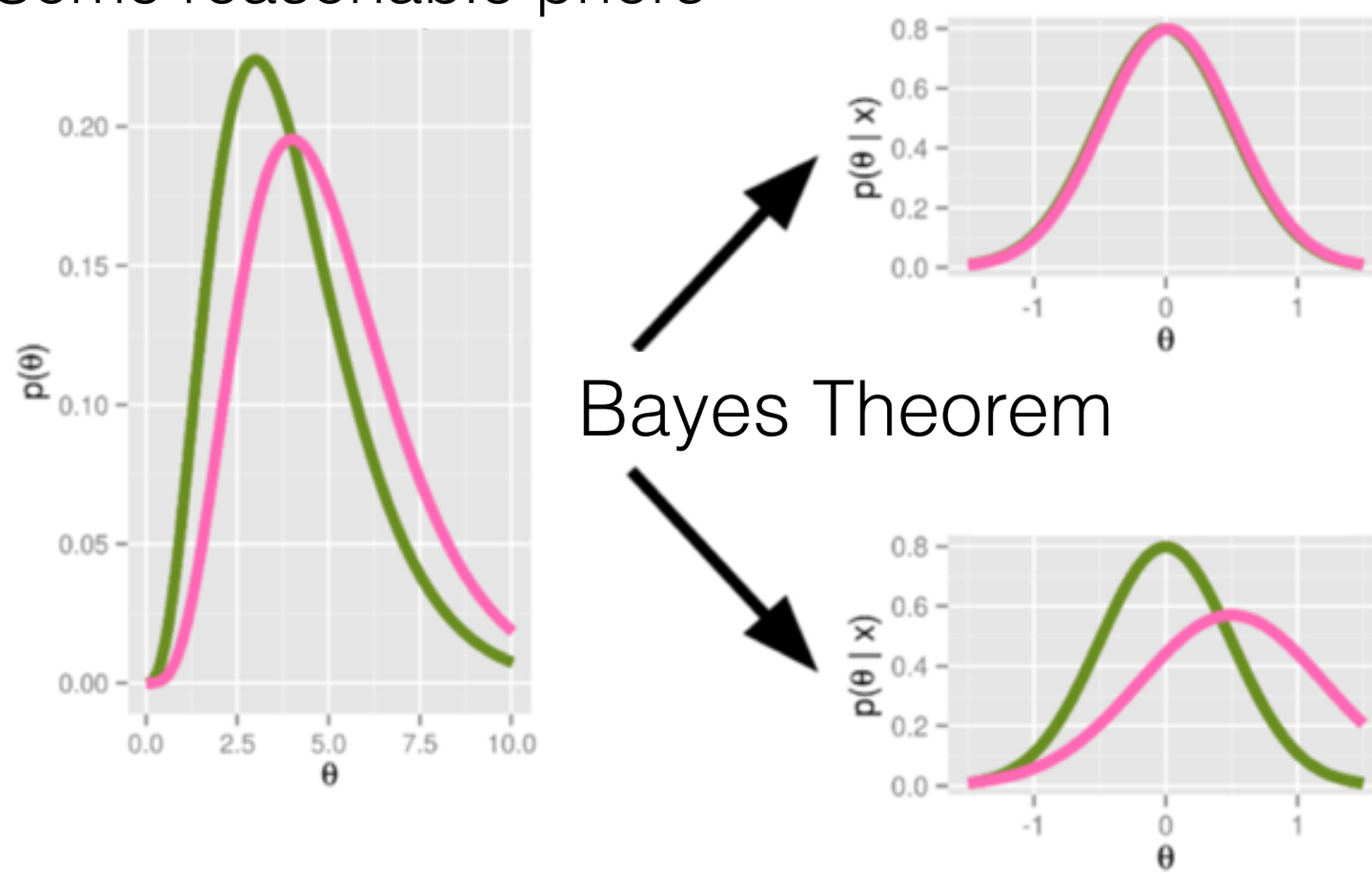

variational Bayes

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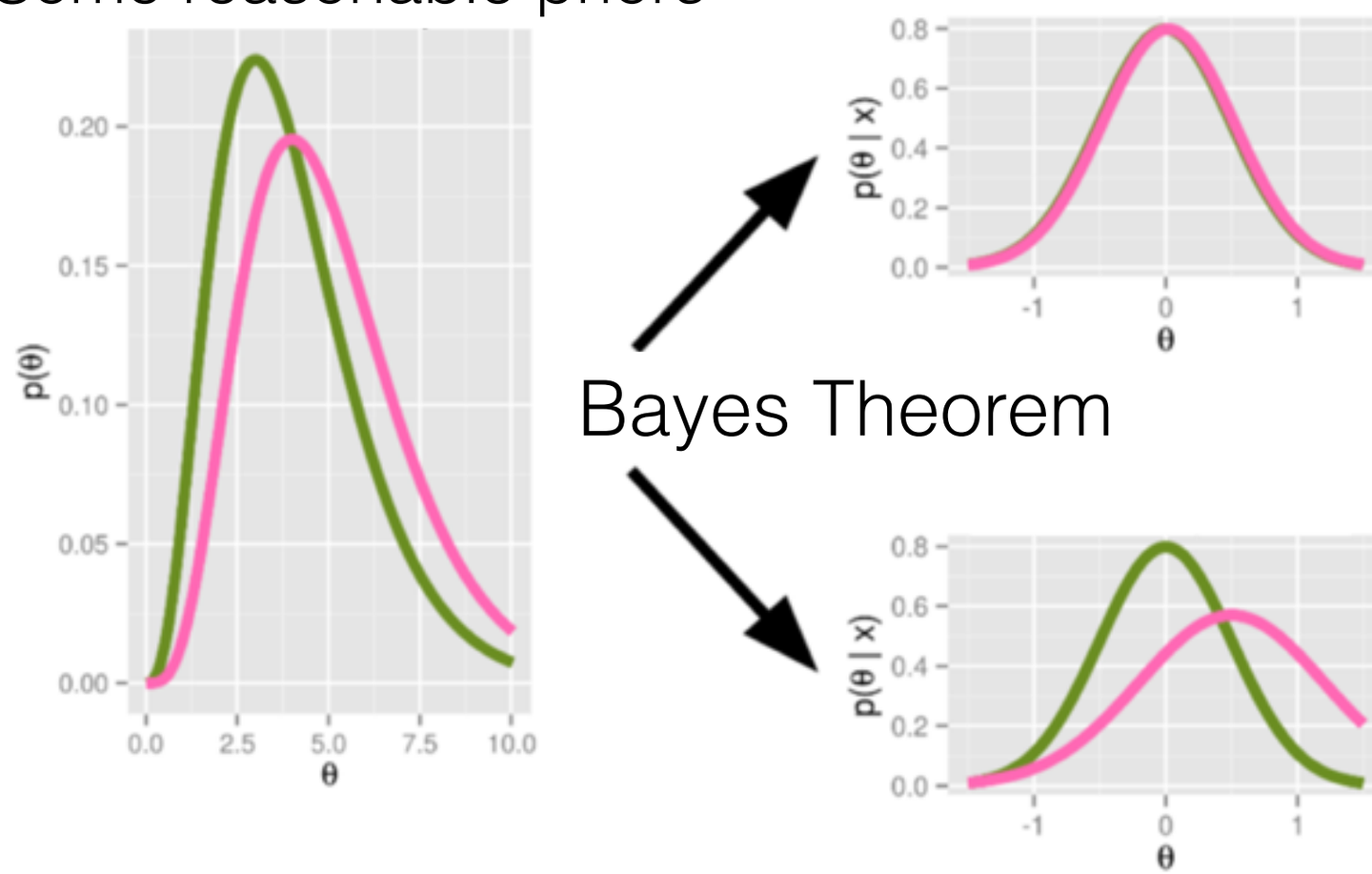

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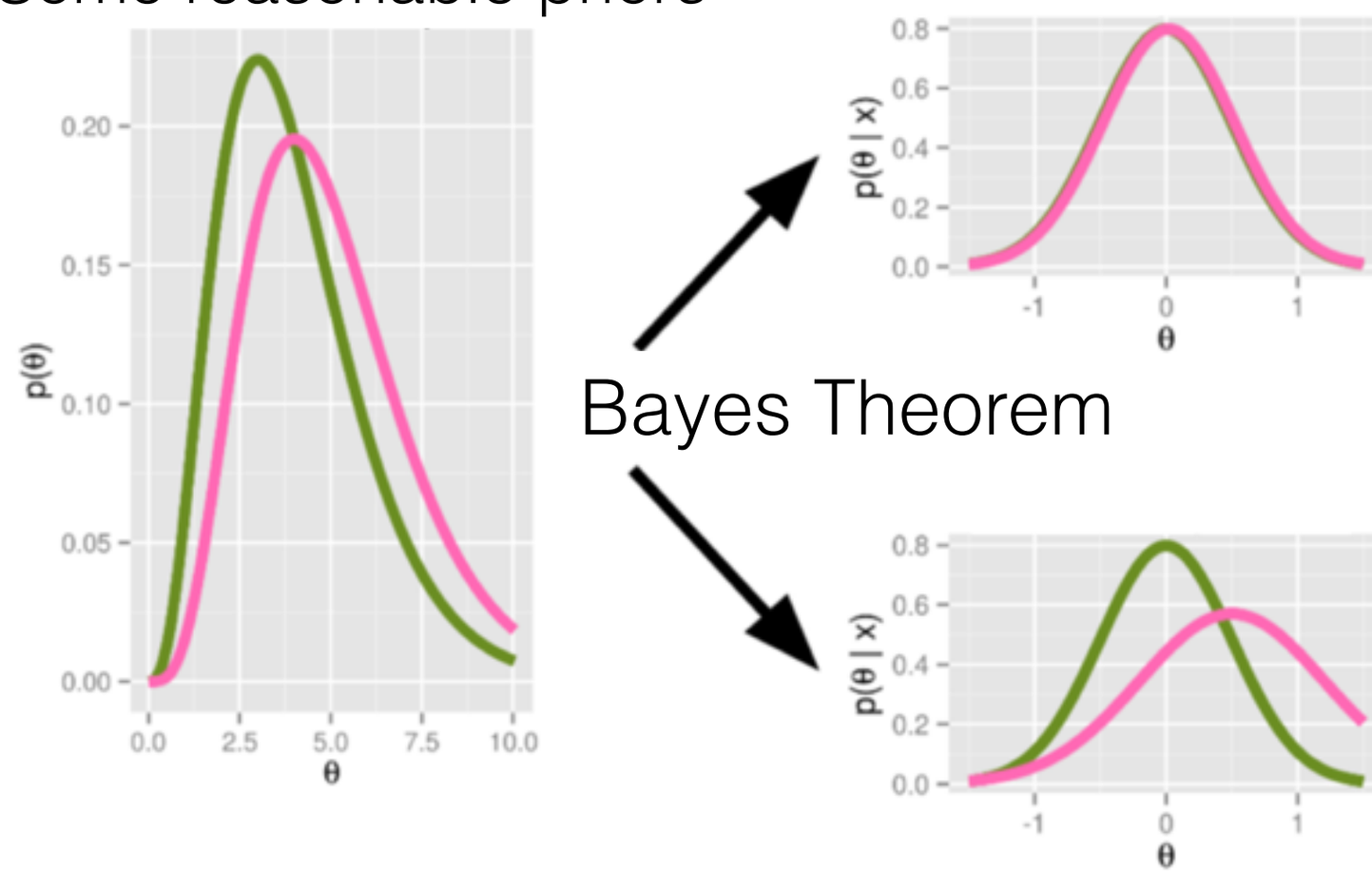

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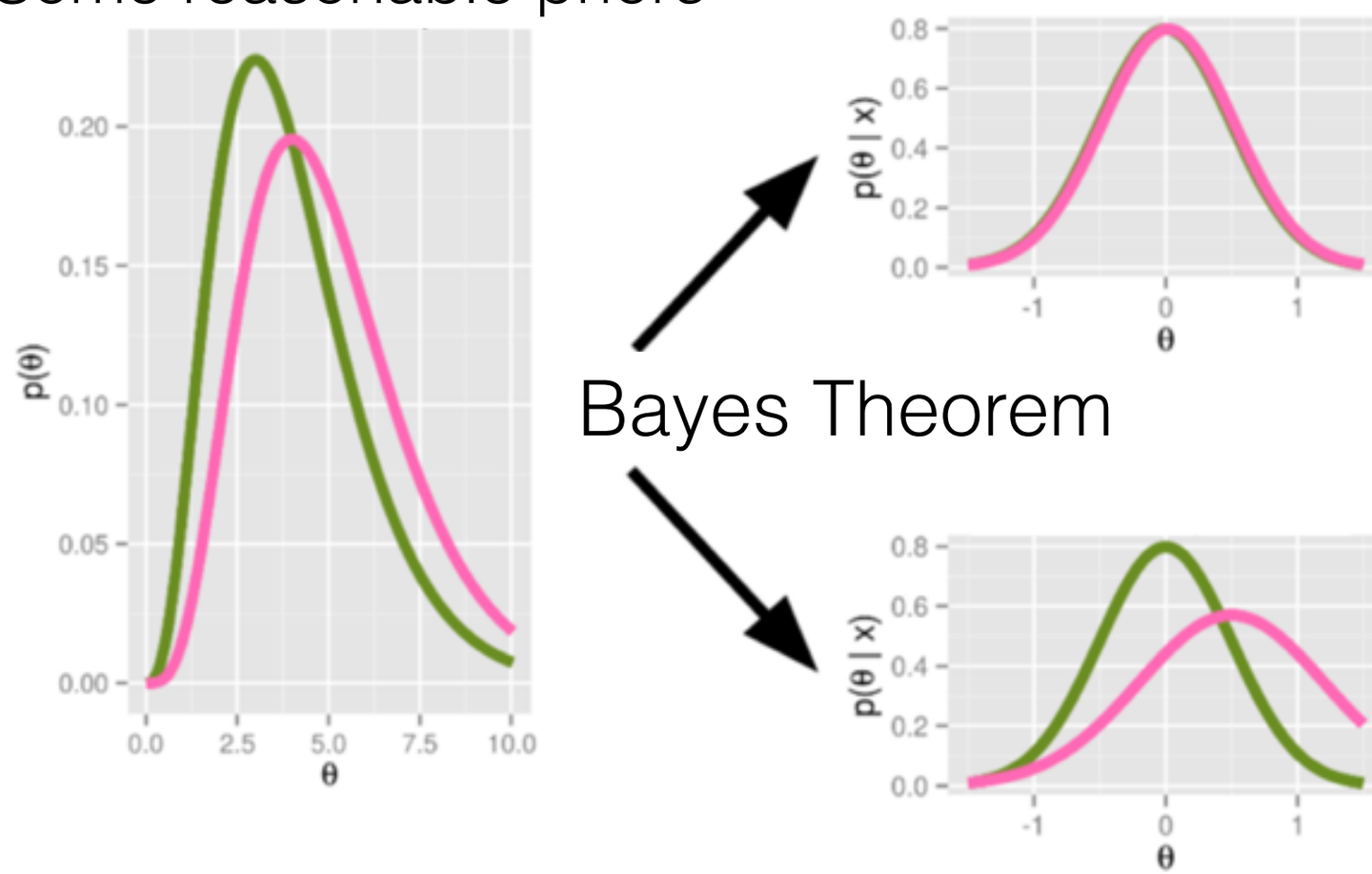

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[see also Opper, Winther 2003]

Roadmap

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- Variational Bayes as an alternative to MCMC

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- Challenges of VB
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- Big idea: derivatives/perturbations are relatively easy in VB

Variational Bayes

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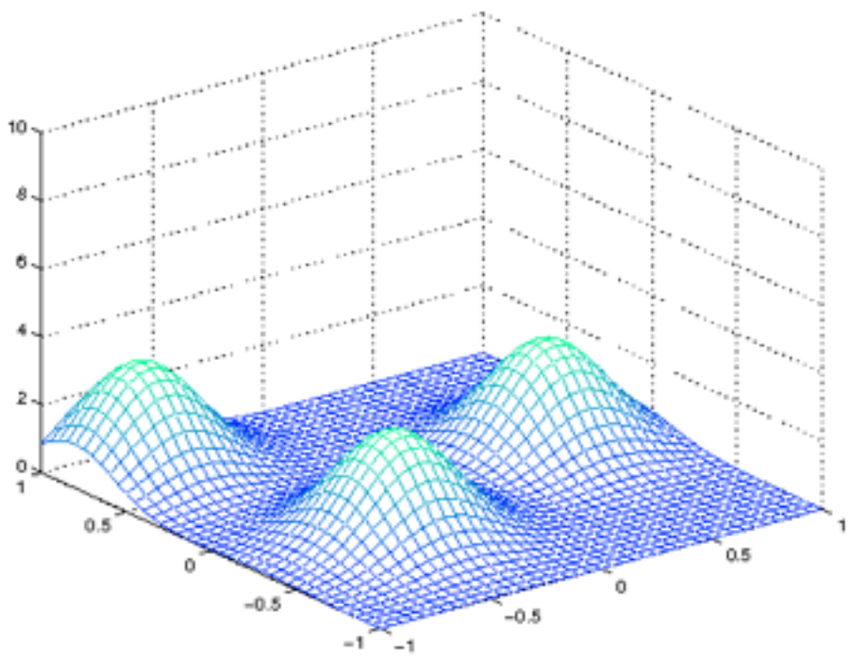
- VB approximation

Variational Bayes



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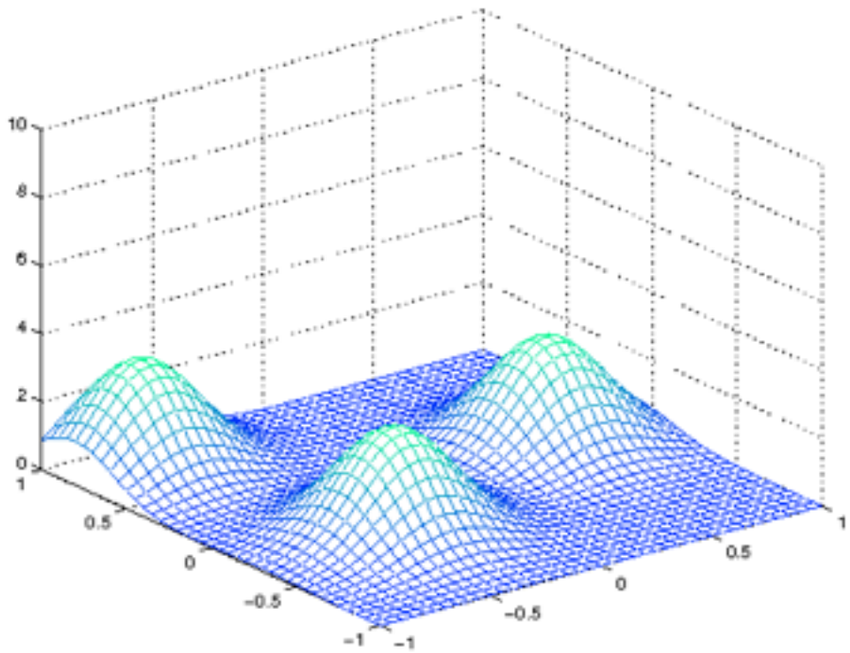

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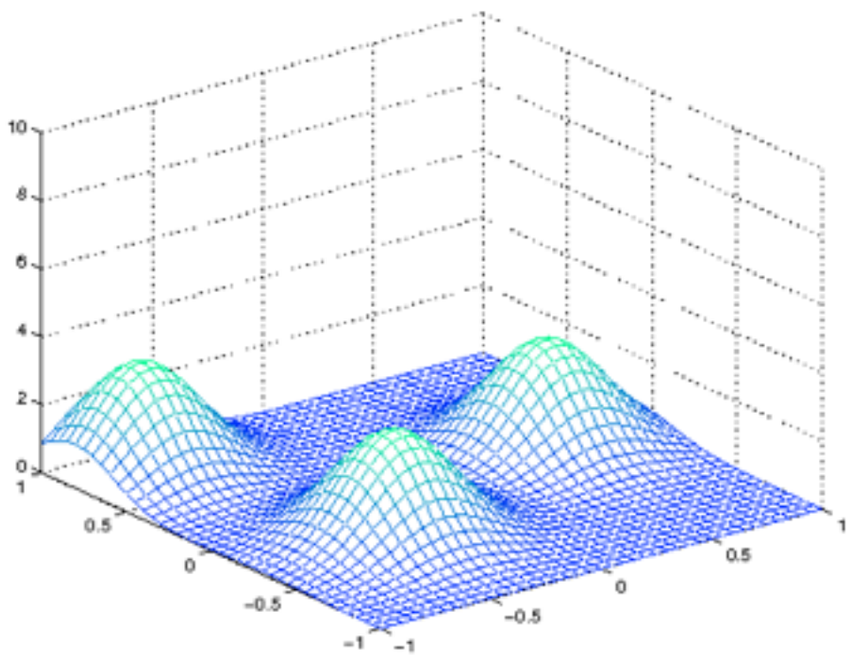

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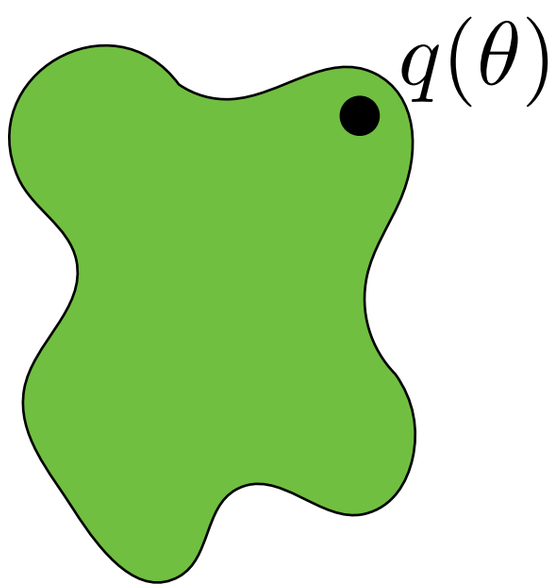

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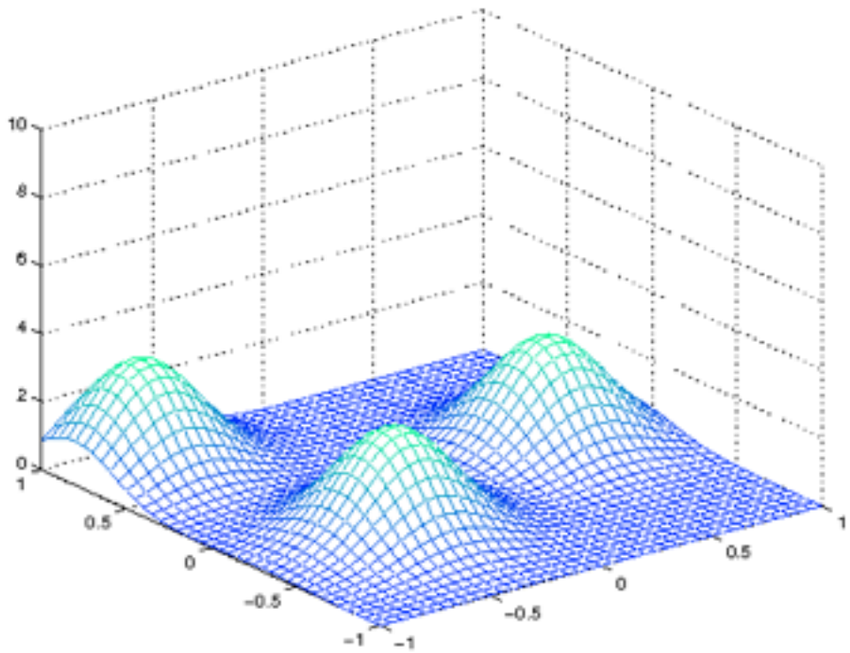

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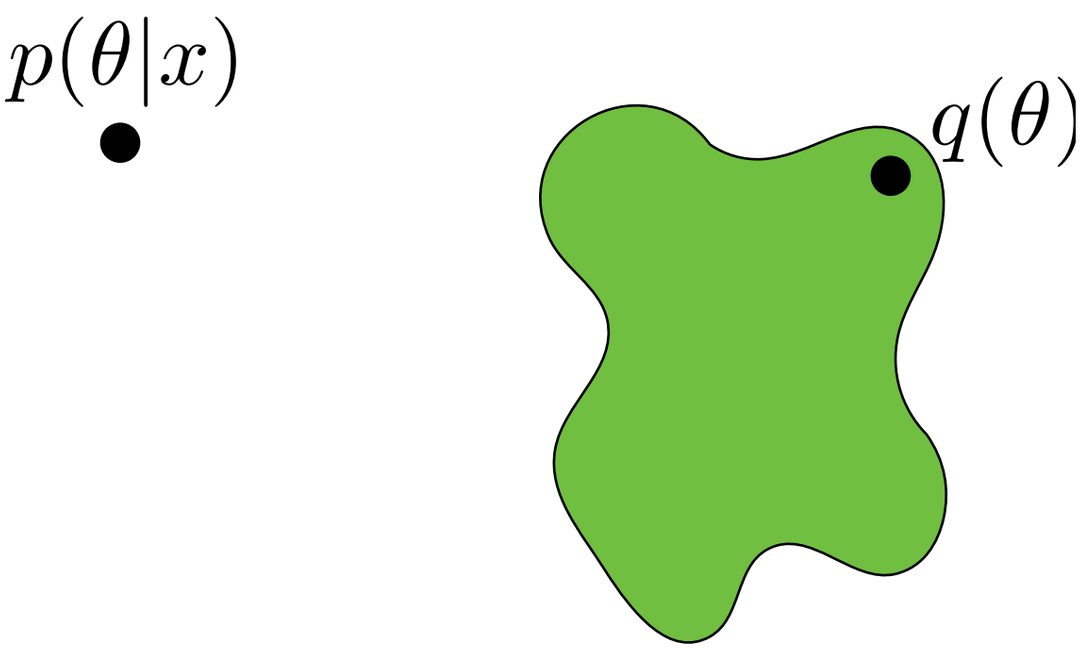

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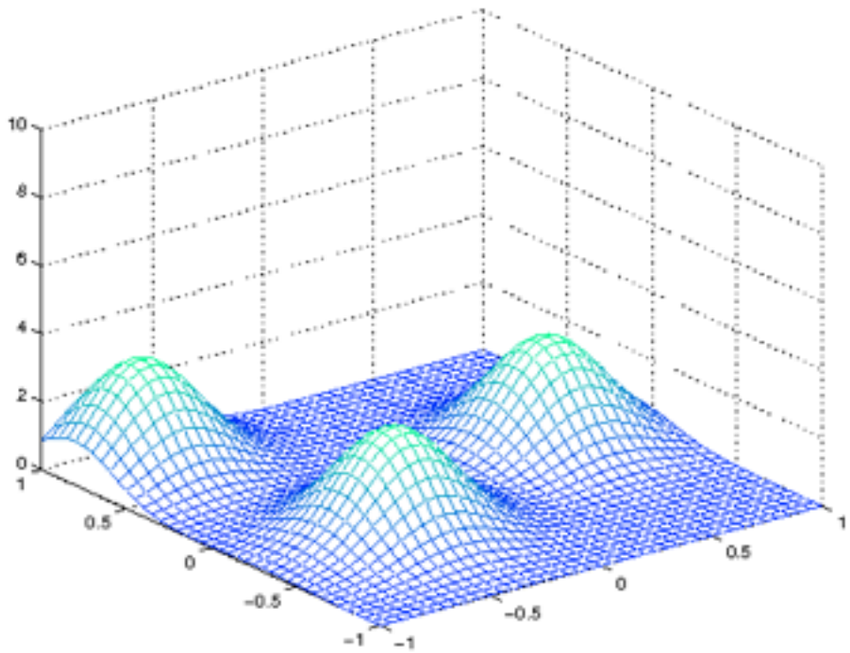

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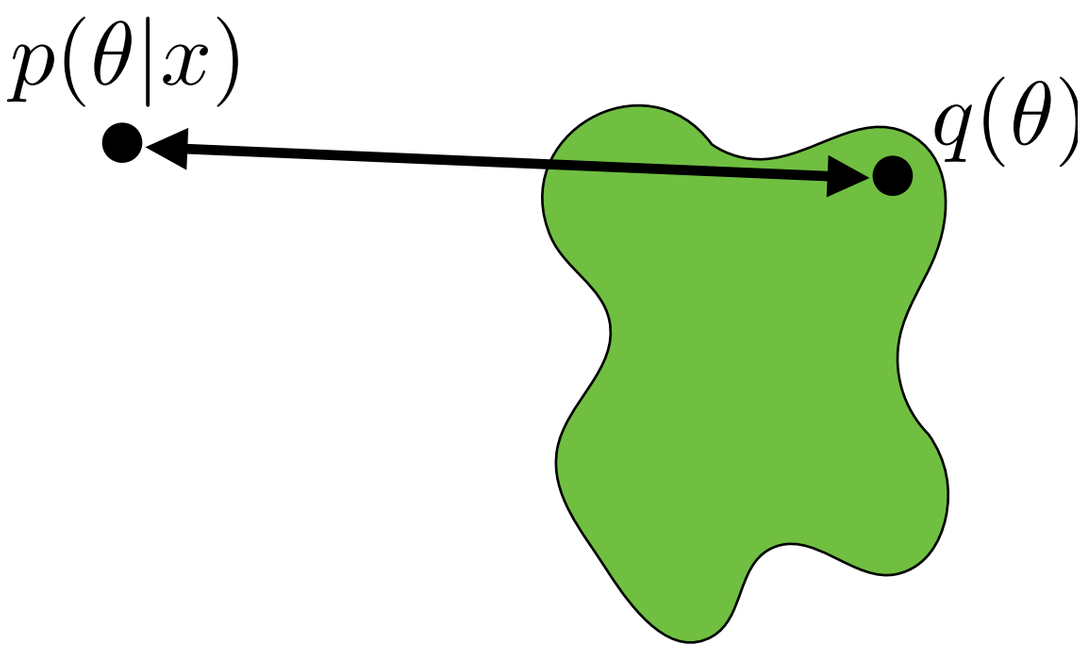

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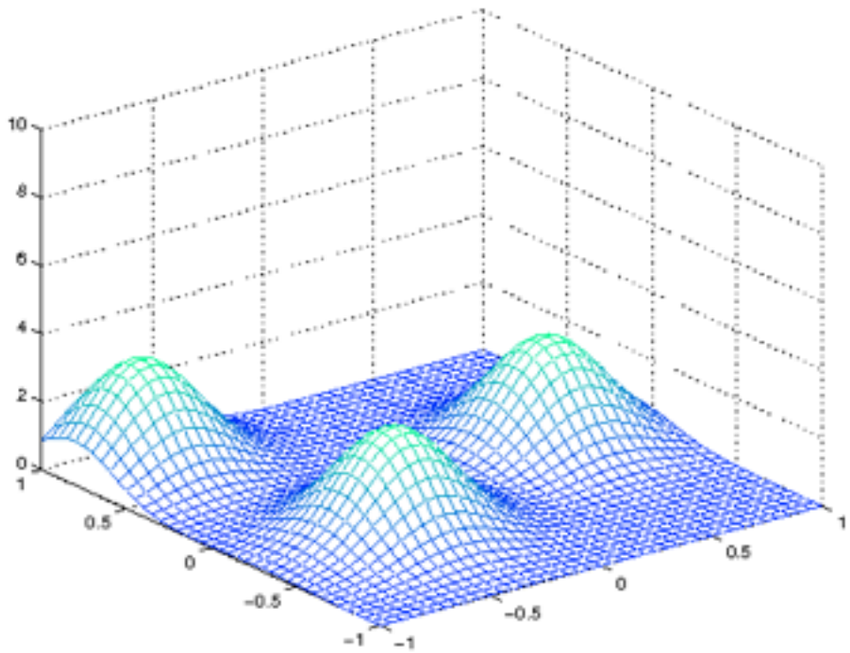

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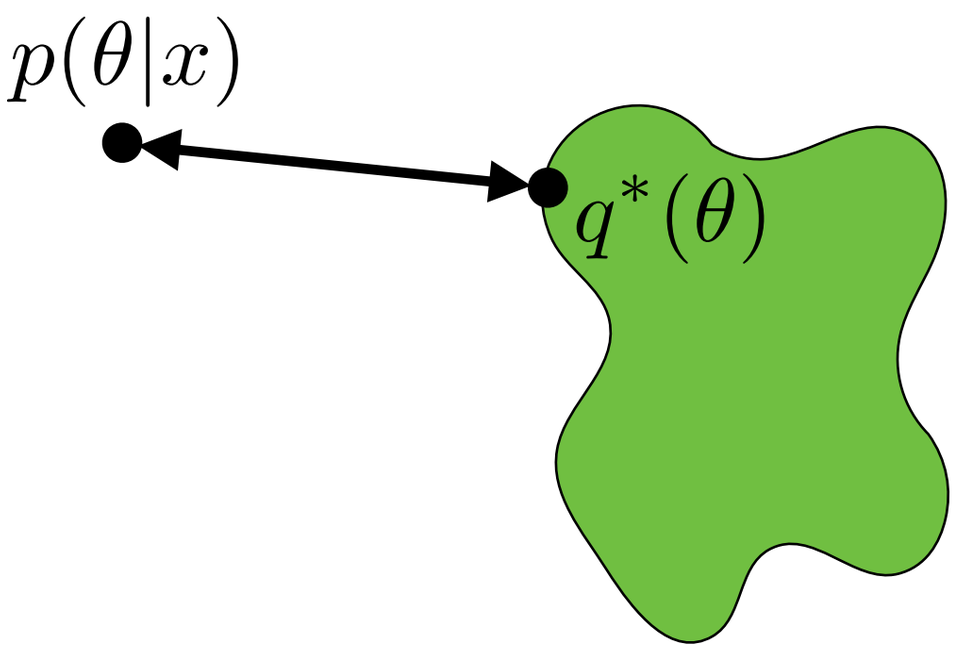

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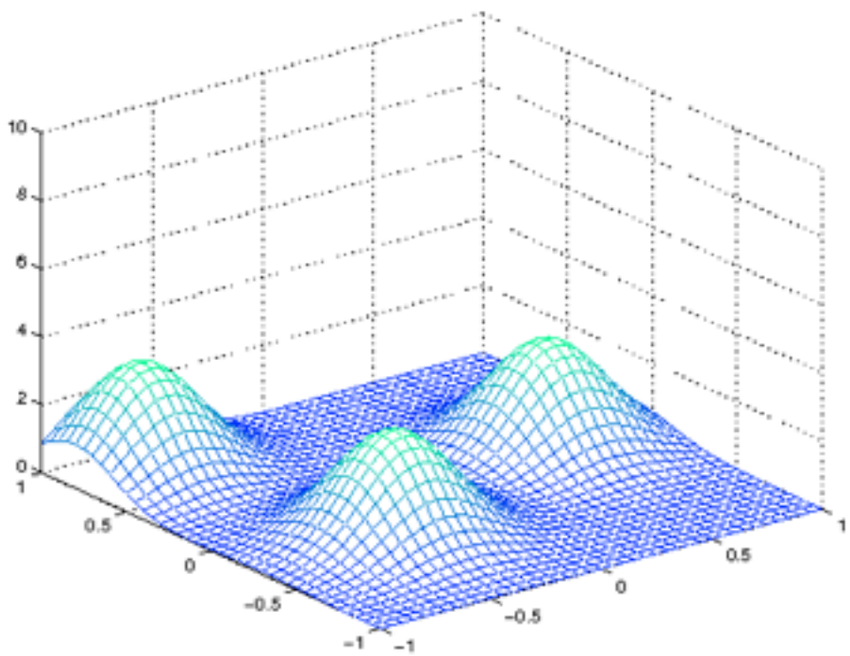

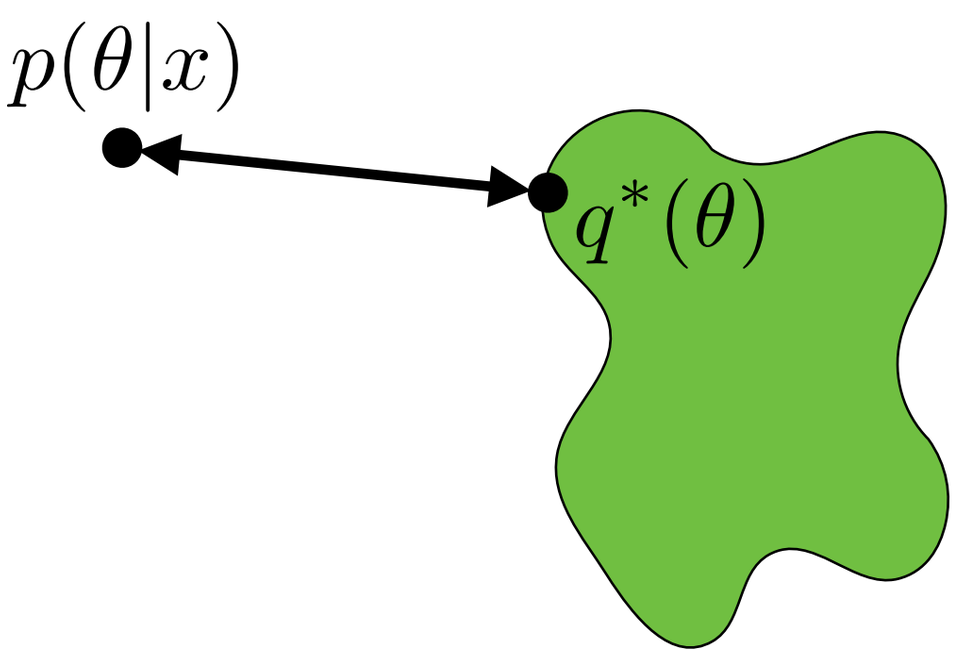

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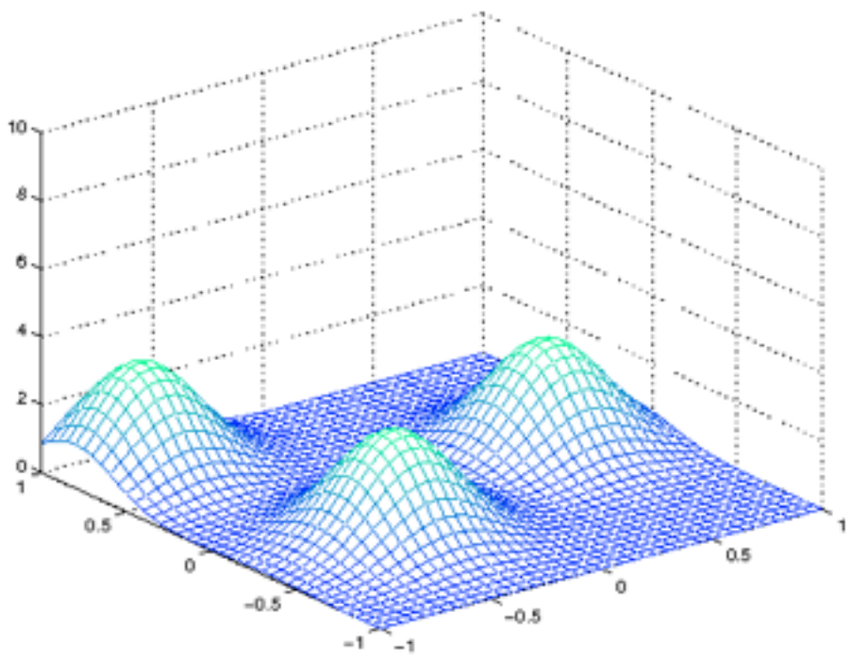

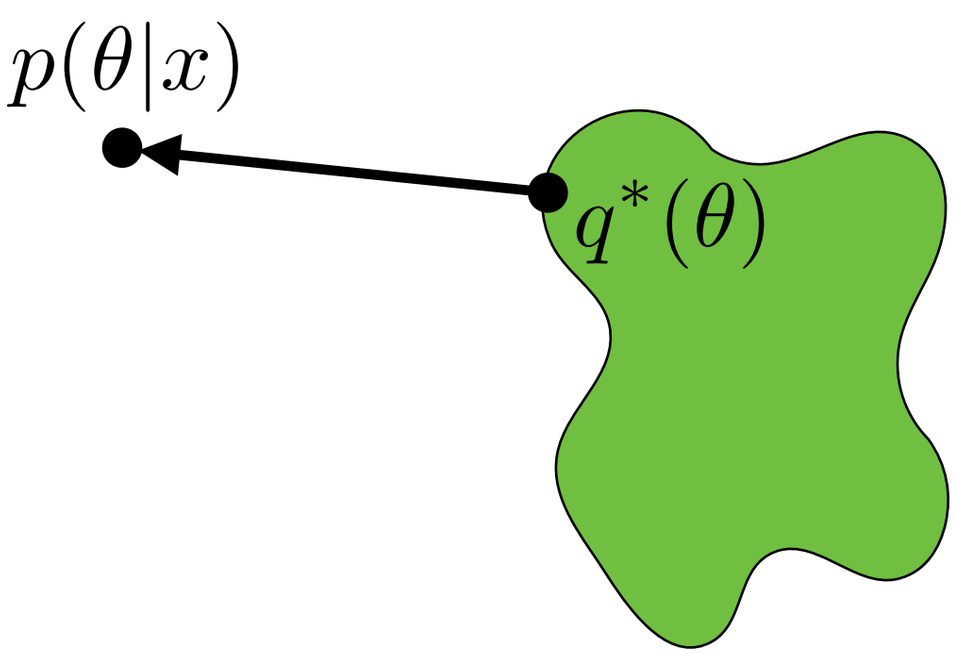

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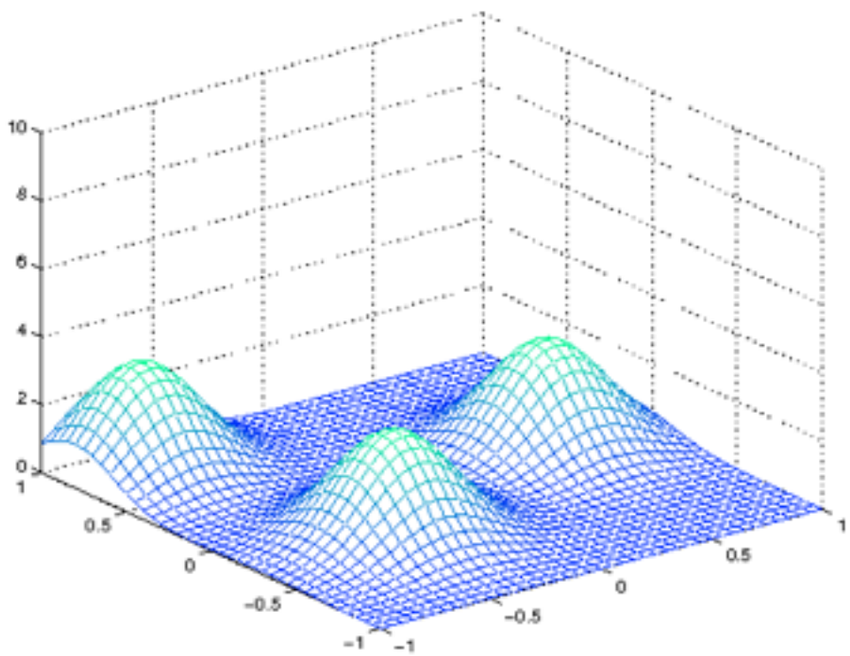

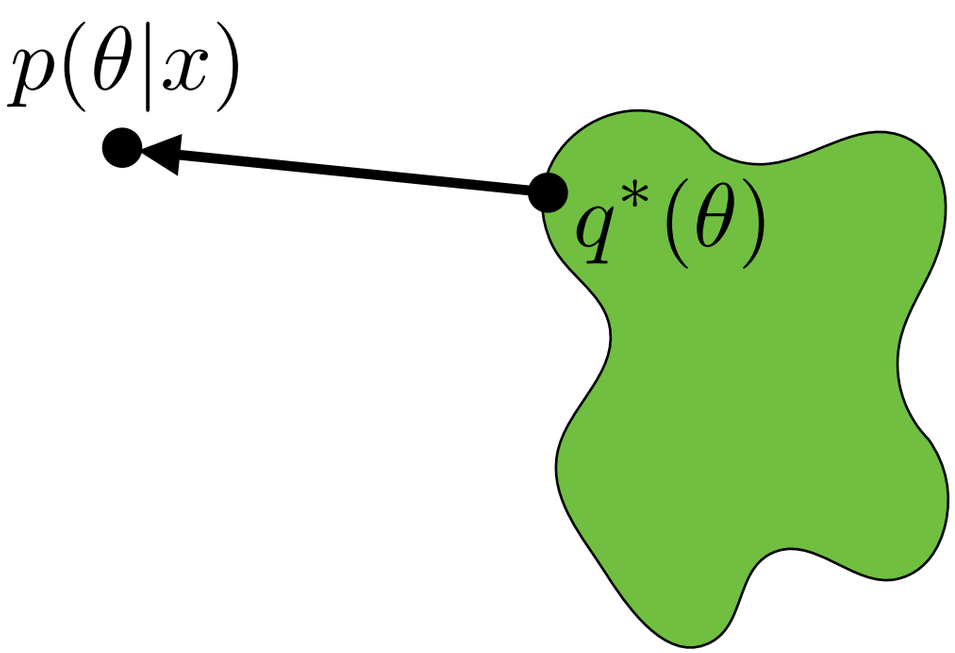

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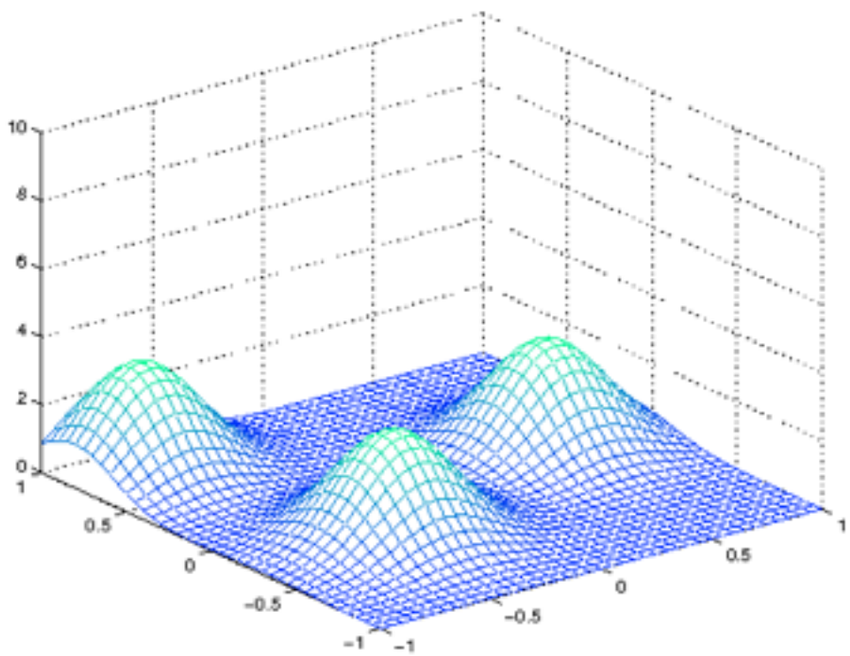

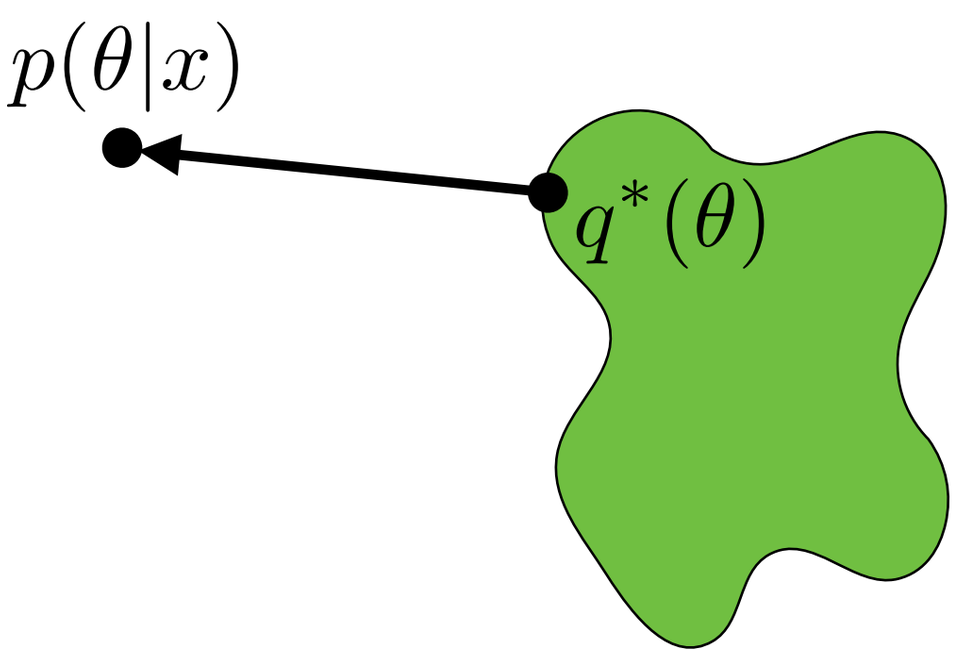

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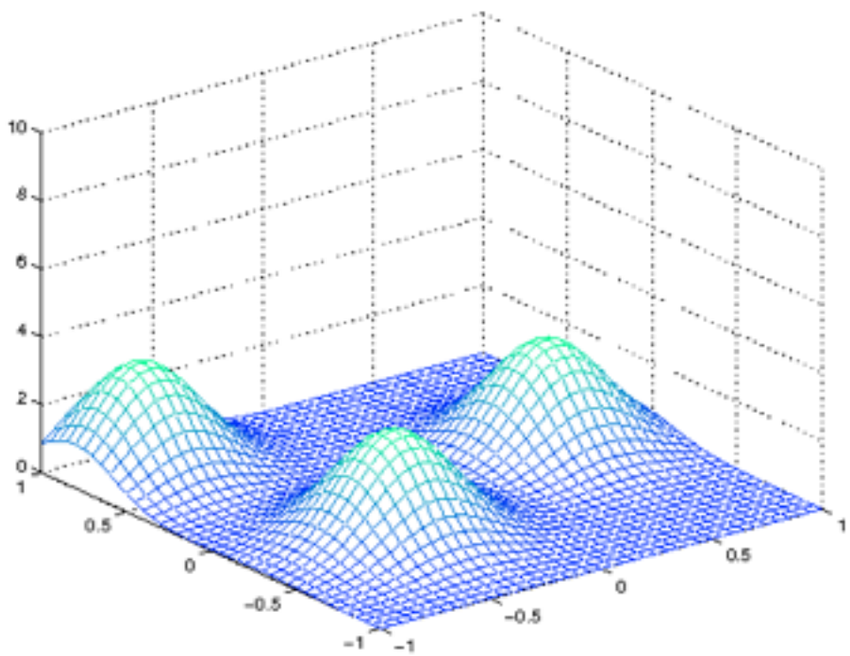
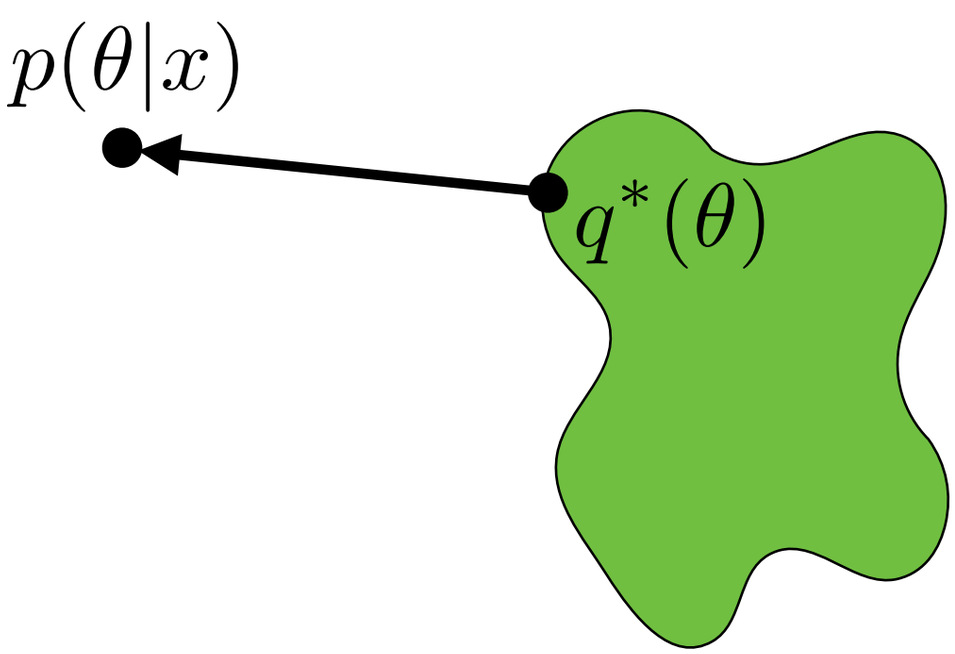
- VB practical success
  - point estimates and prediction

Variational Bayes





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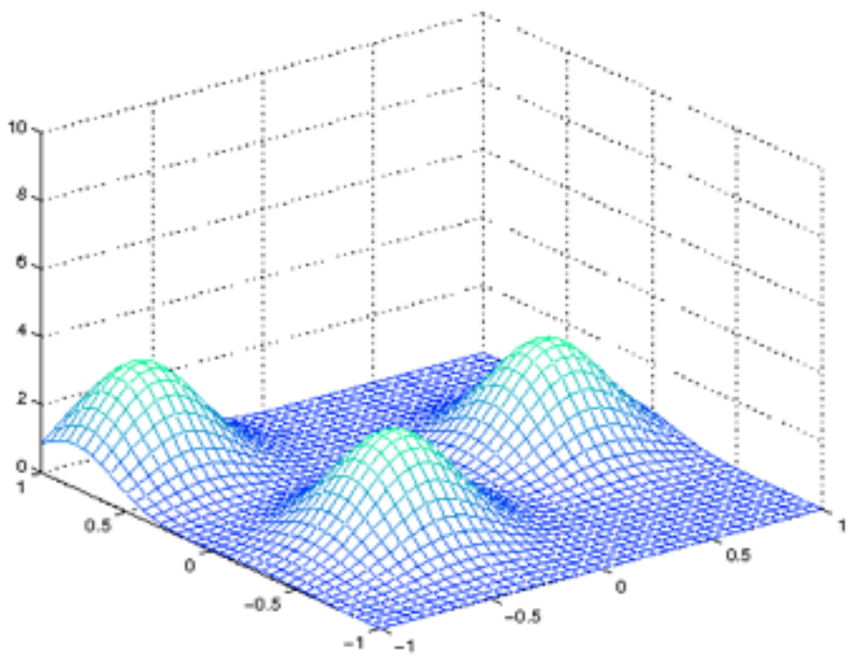
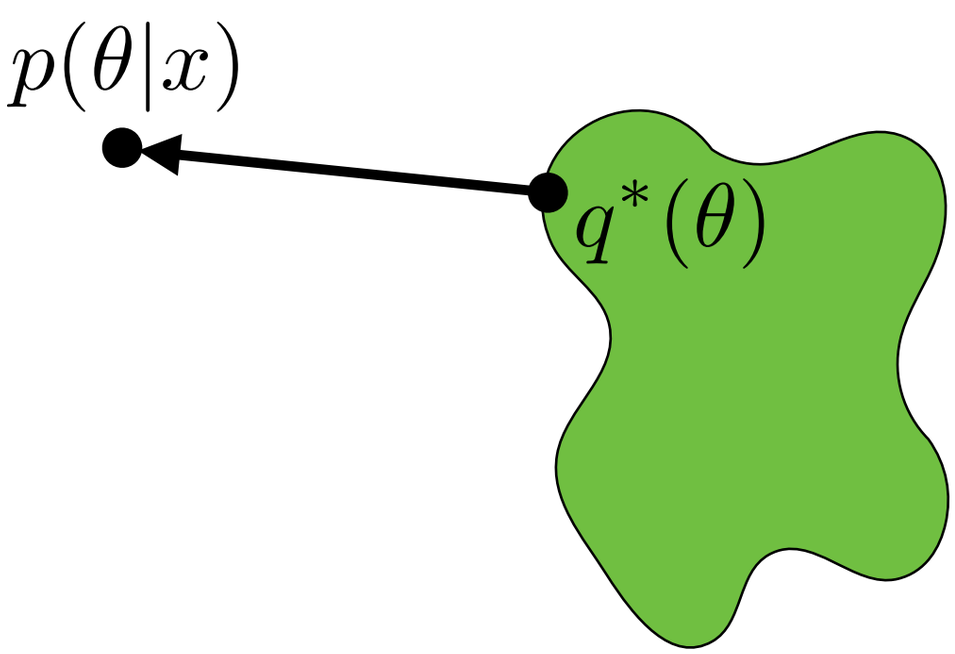
  - point estimates and prediction
  - fast

[Broderick, Boyd, Wibisono, Wilson, Jordan 2013]

Variational Bayes





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- VB practical success
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  - fast, streaming, distributed

[Broderick, Boyd, Wibisono, Wilson, Jordan 2013]

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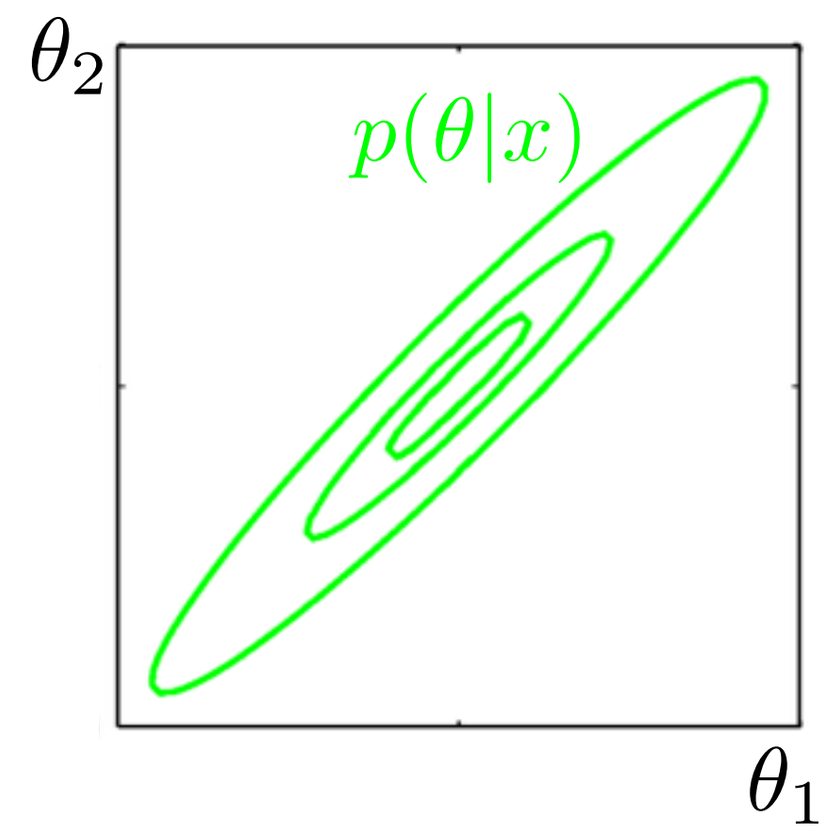

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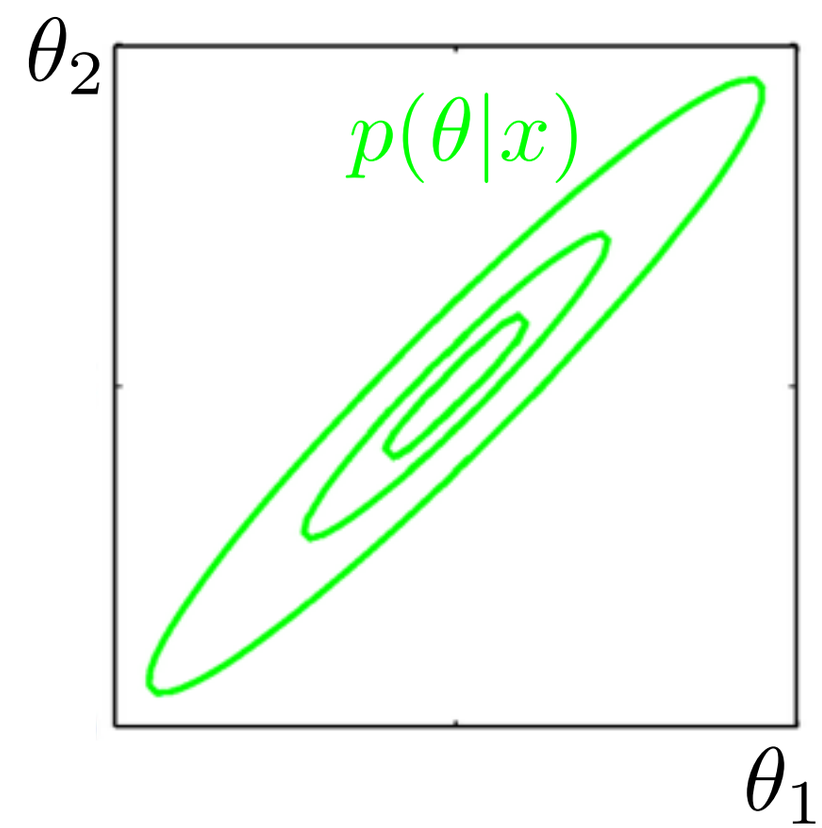

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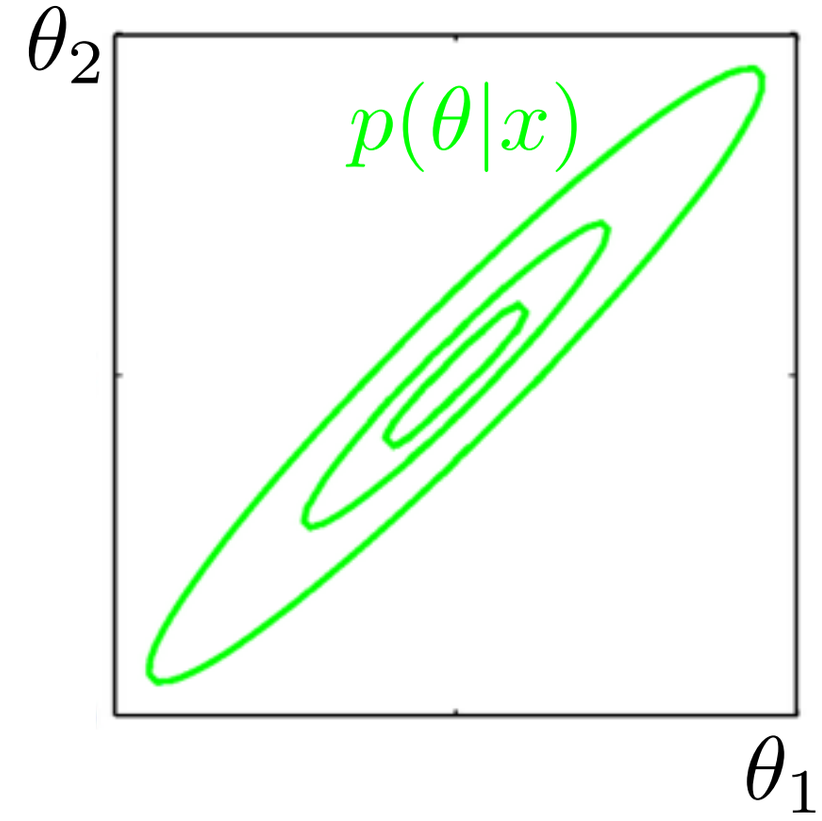

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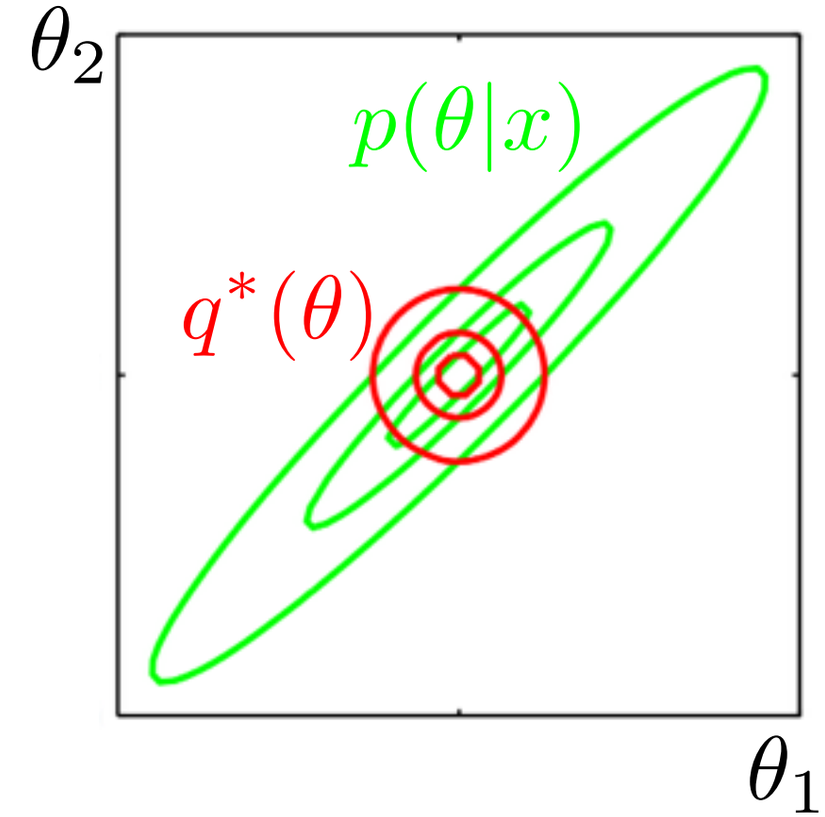

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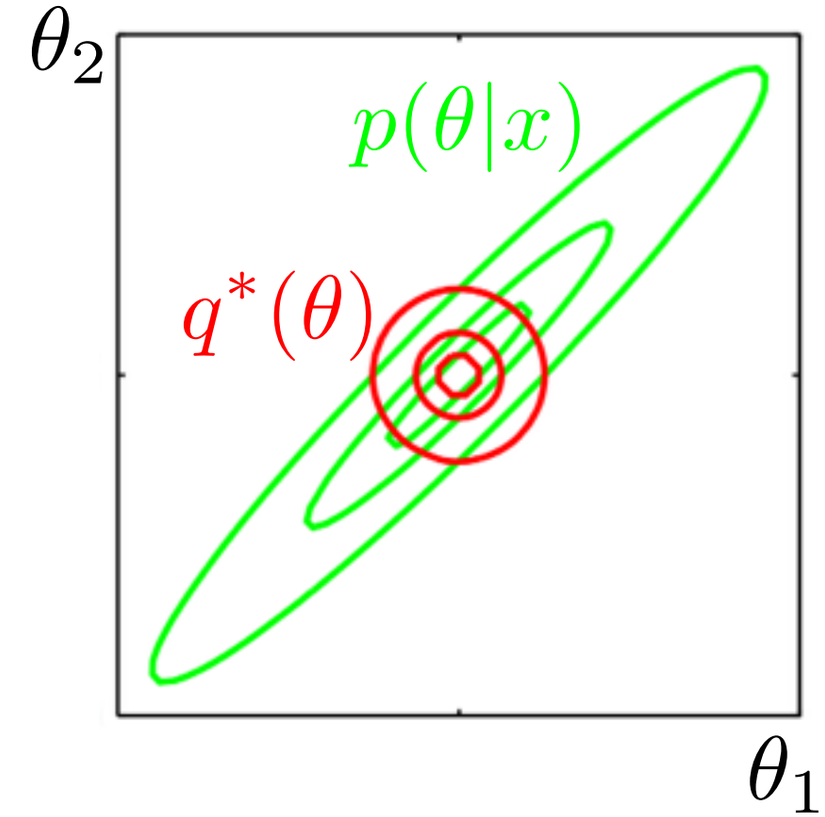

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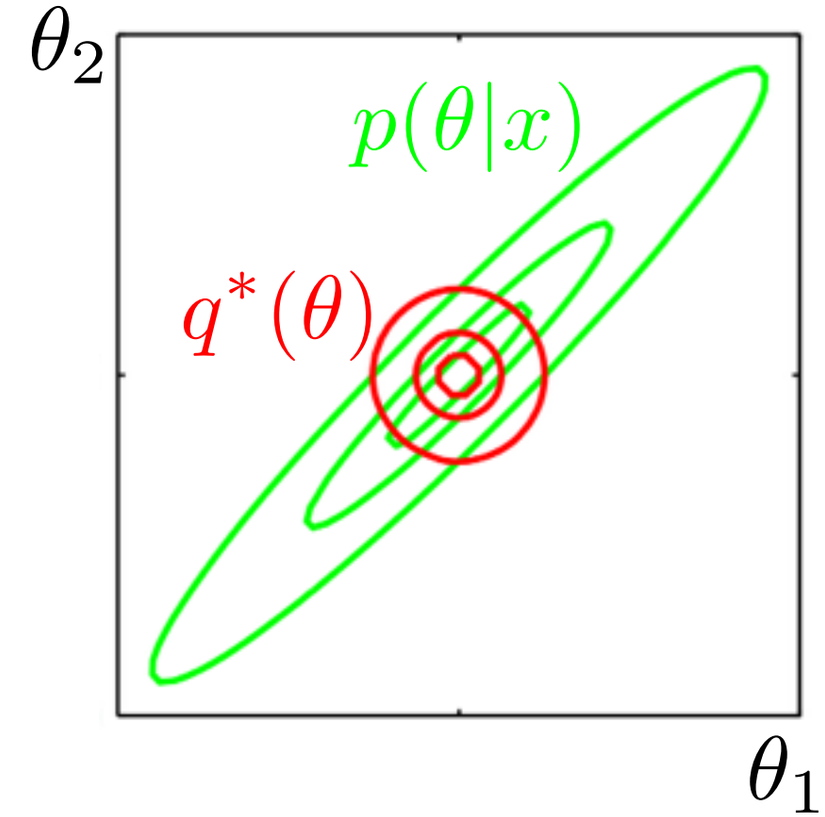

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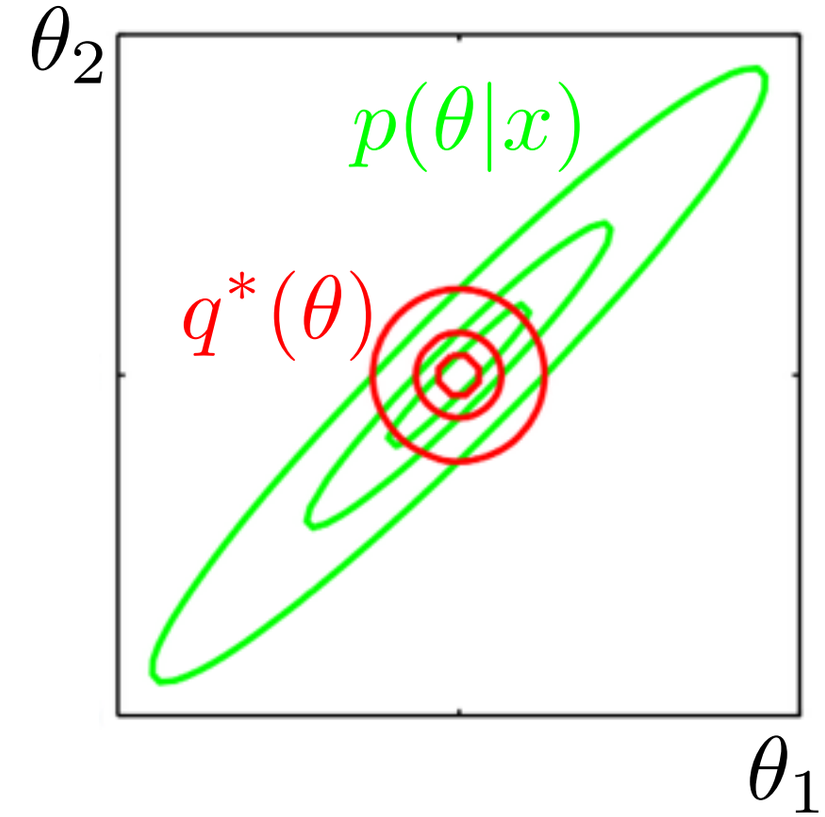

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[MacKay 2003; Bishop 2006; Wang, Titterington 2004; Turner, Sahani 2011]

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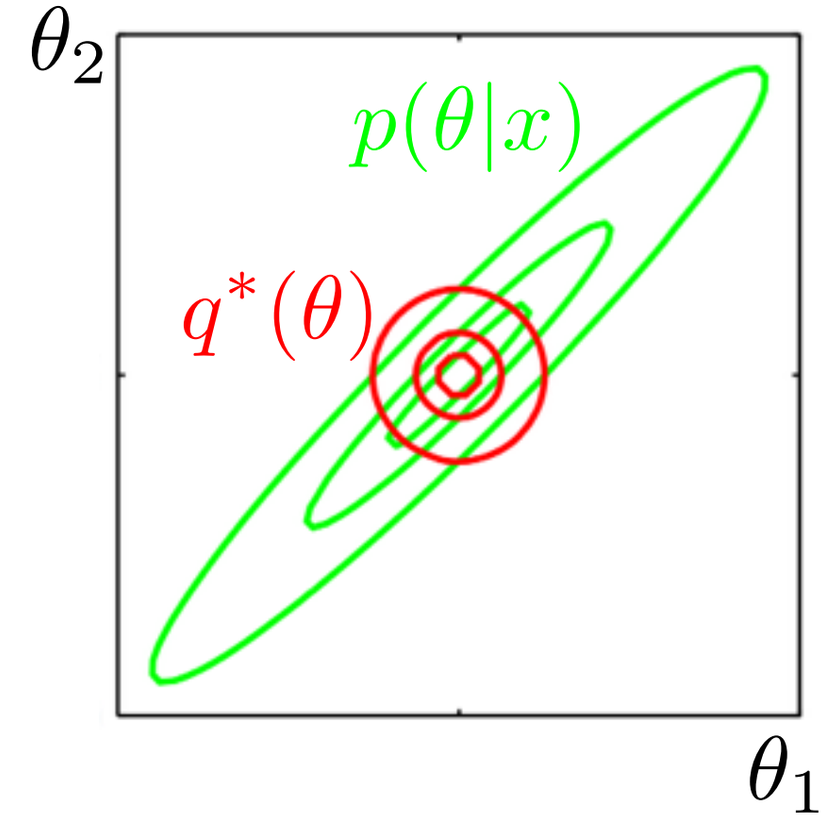

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[MacKay 2003; Bishop 2006; Wang, Titterington 2004; Turner, Sahani 2011]
[Fosdick 2013; Dunson 2014; Bardenet, Doucet, Holmes 2015]

Linear response

[see also Opper, Winther 2003]

Linear response

- Cumulant-generating function

[see also Opper, Winther 2003]

Linear response

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$$C(t) := \log \mathbb{E} e^{t^T \theta}$$

[see also Opper, Winther 2003]

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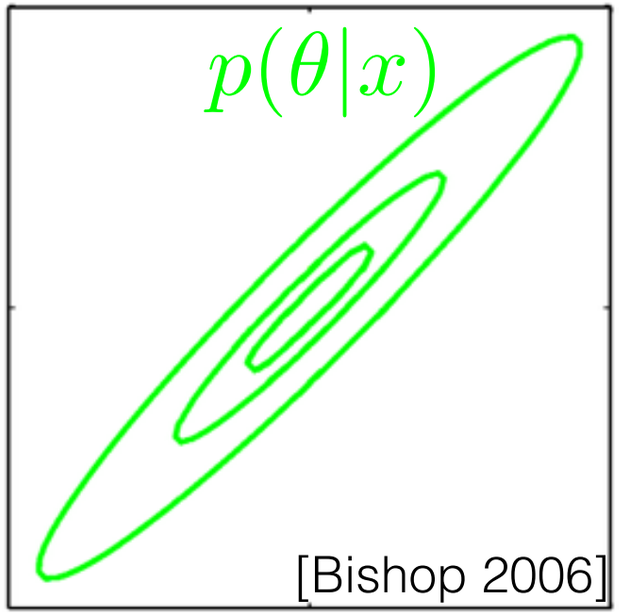

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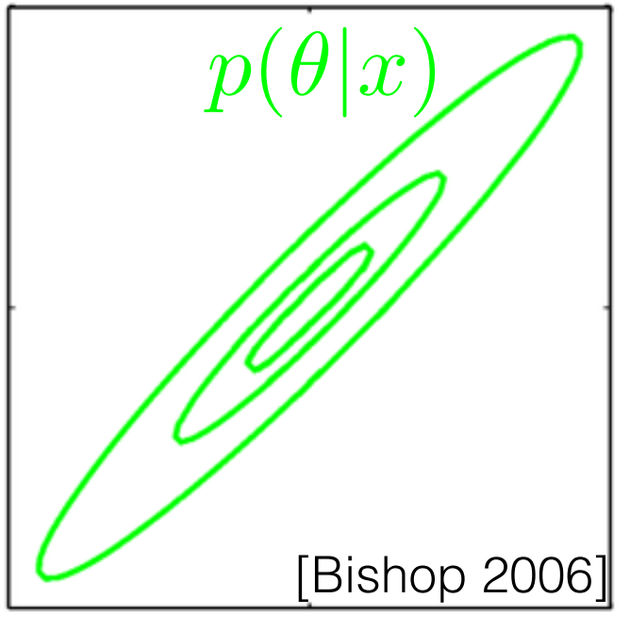

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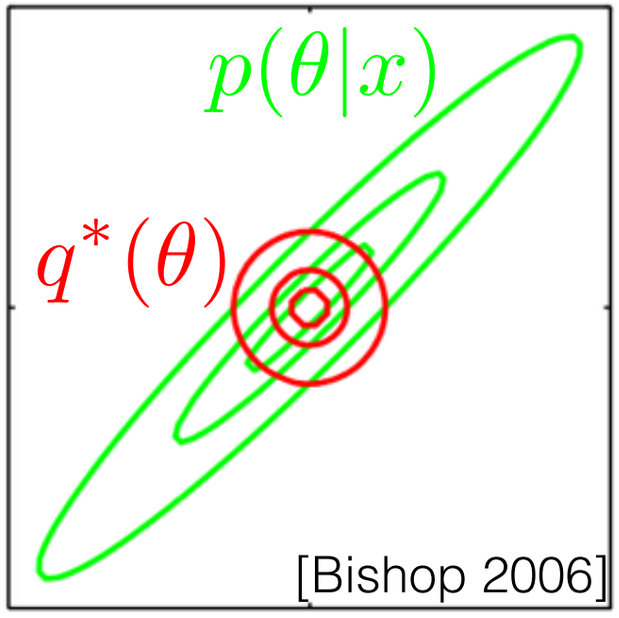

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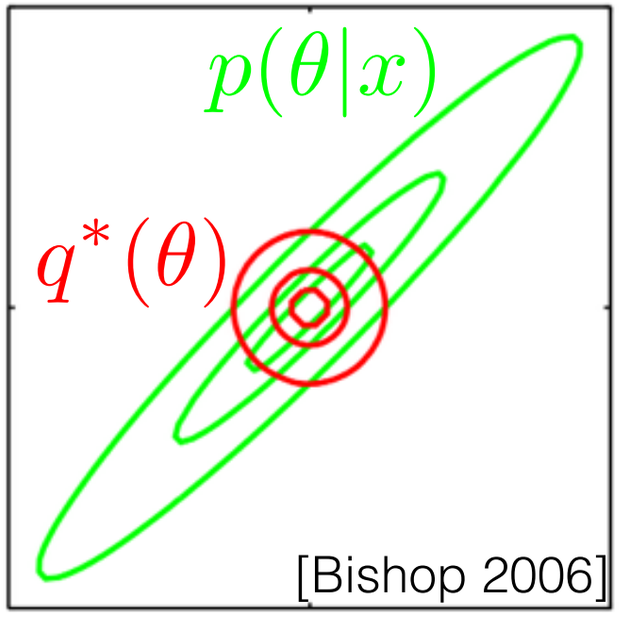

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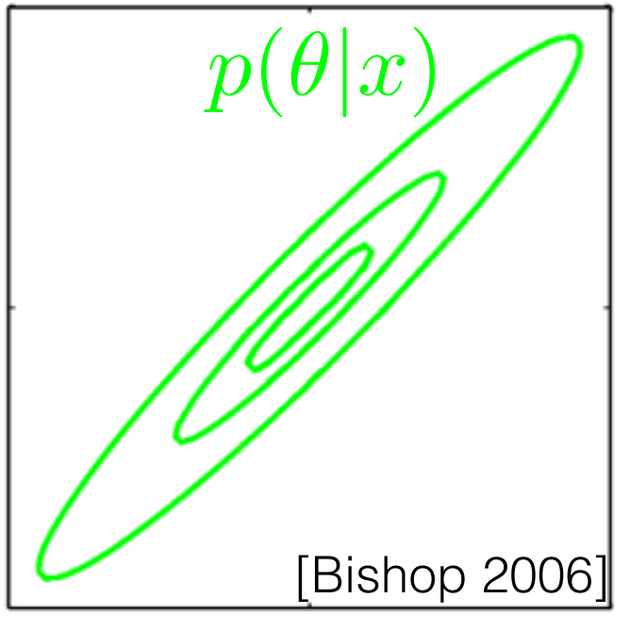

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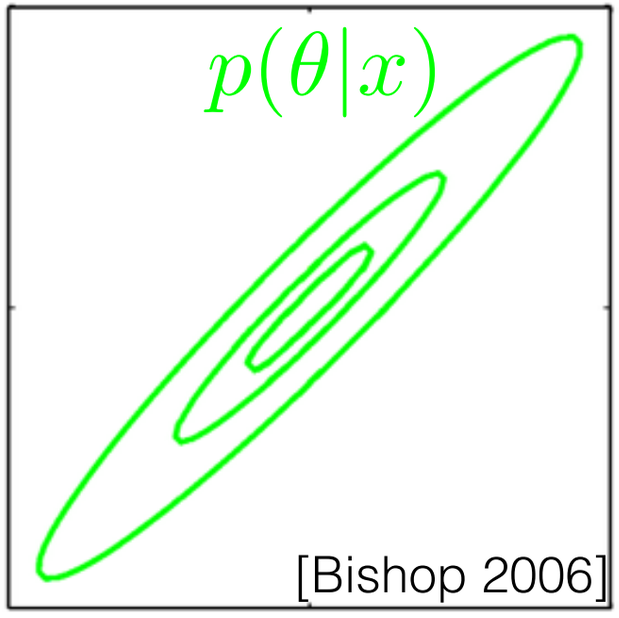

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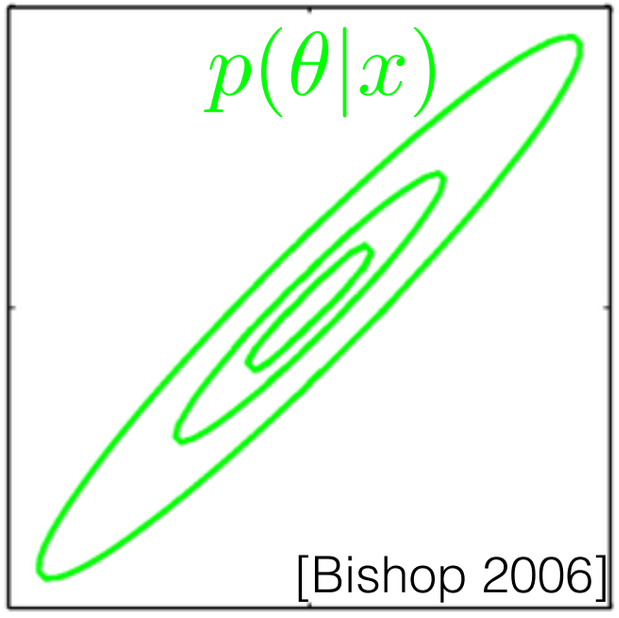

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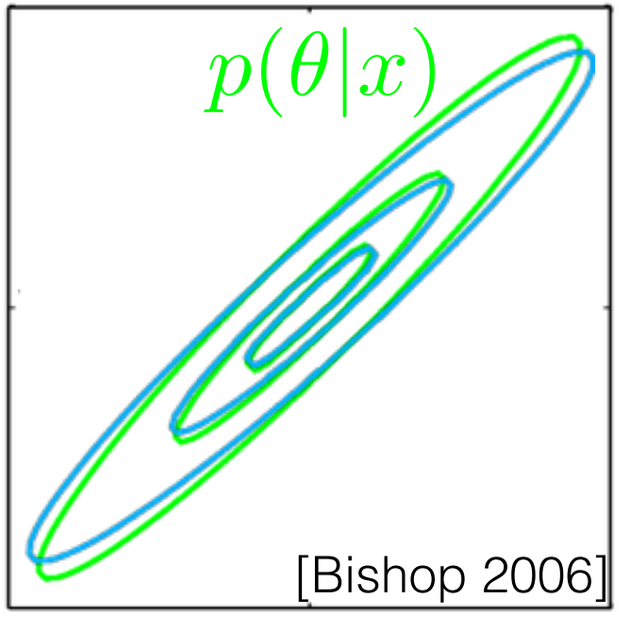

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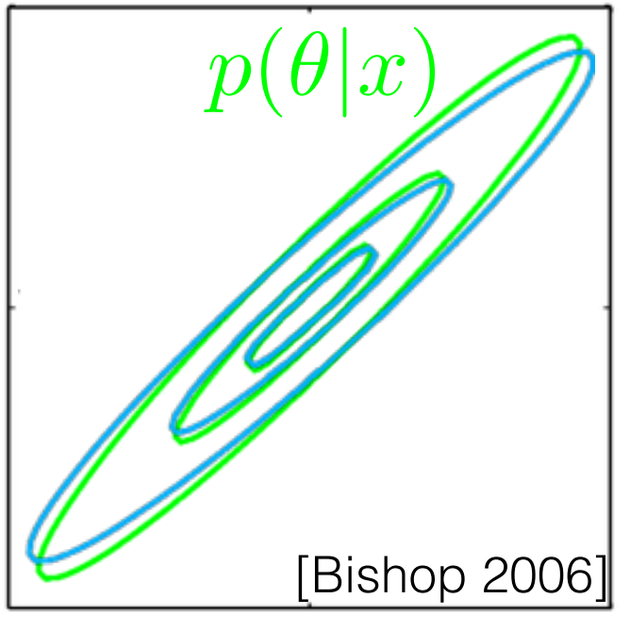

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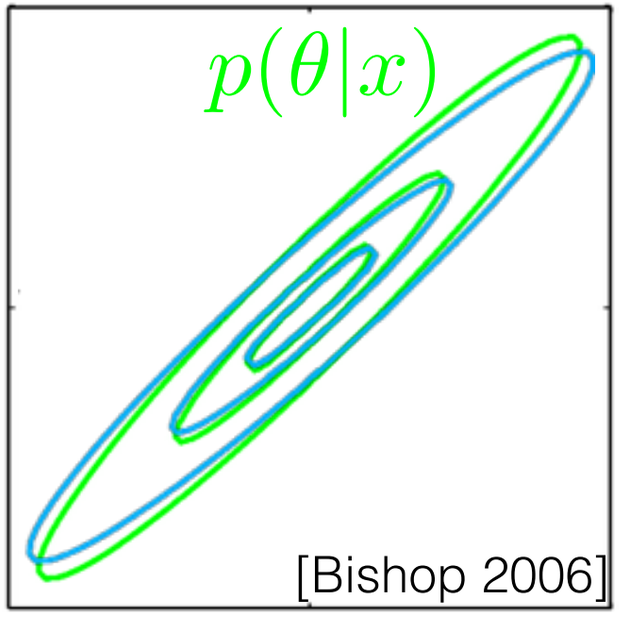

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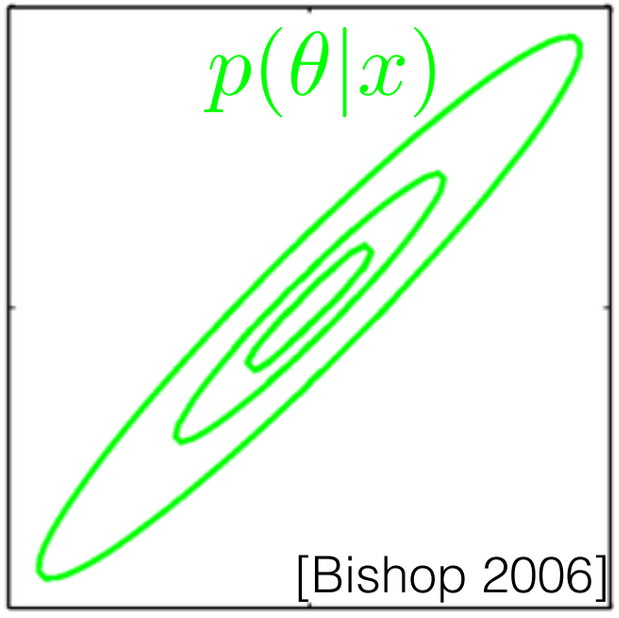

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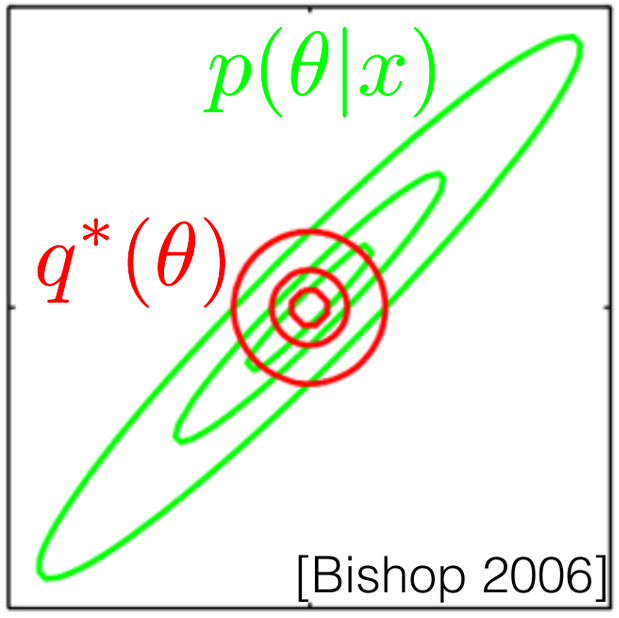

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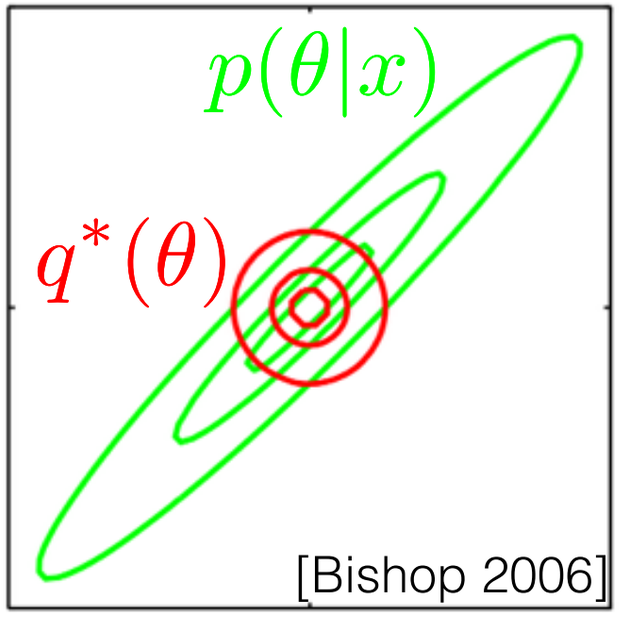

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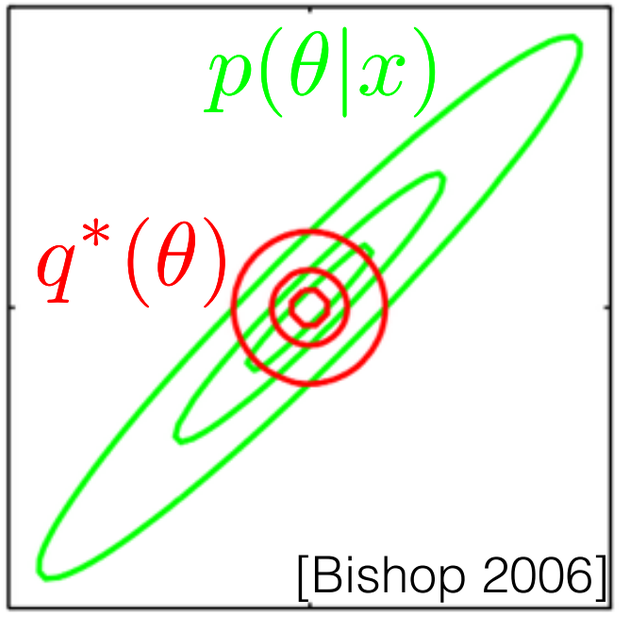

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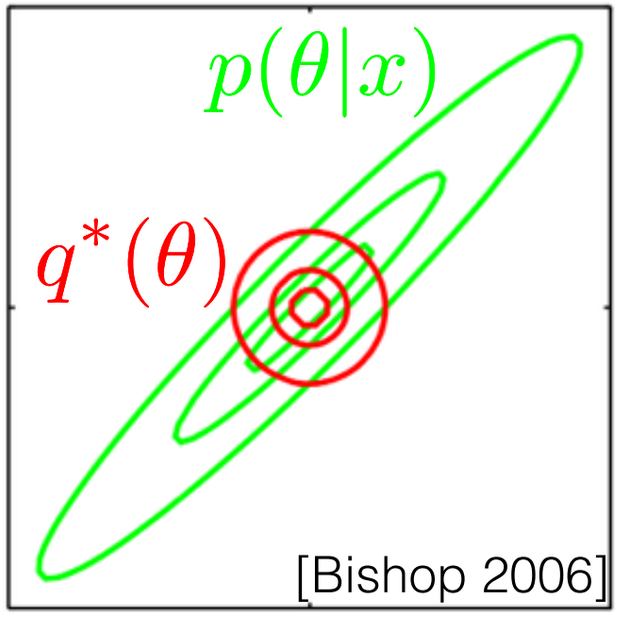

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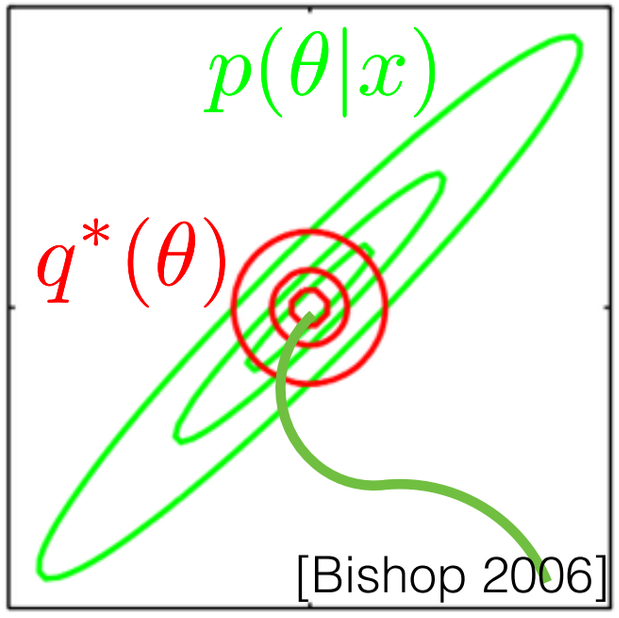

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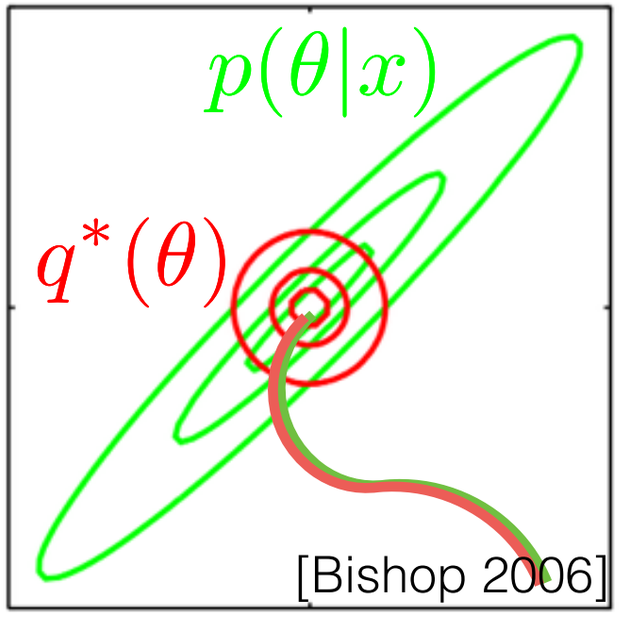

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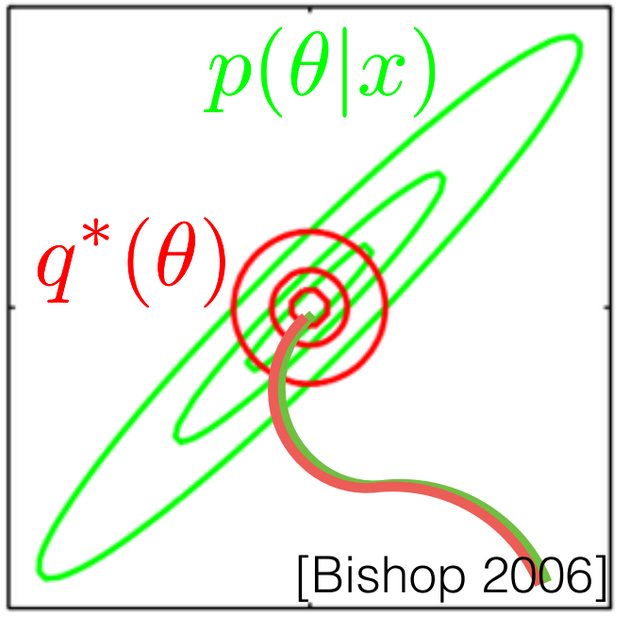

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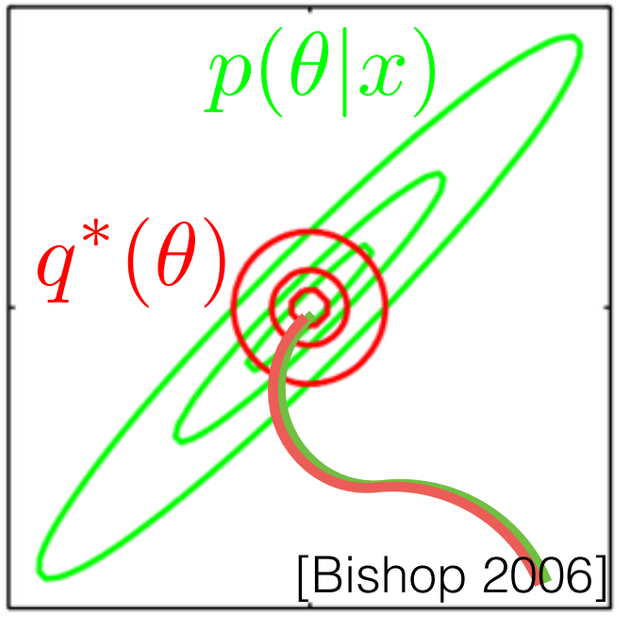

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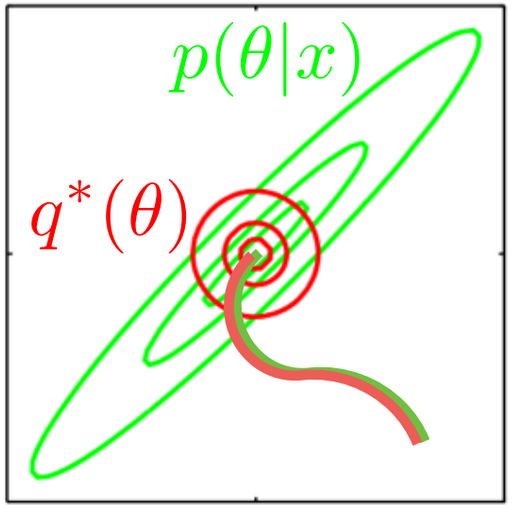

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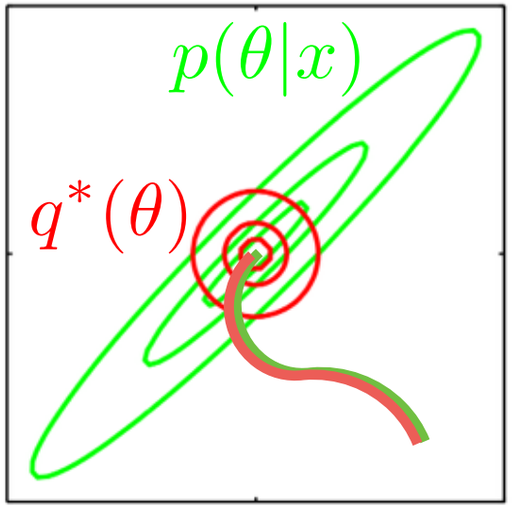

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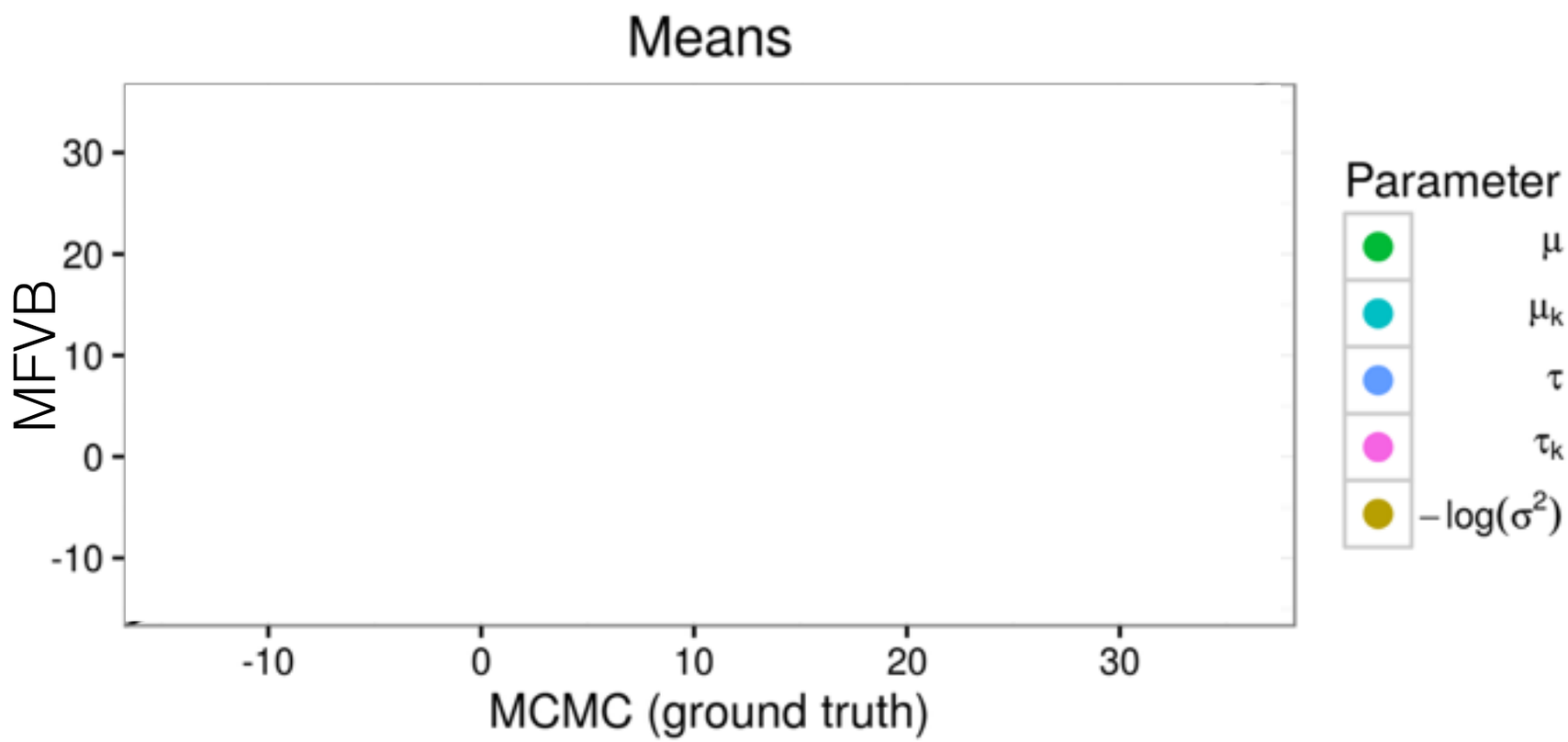
Microcredit Experiment



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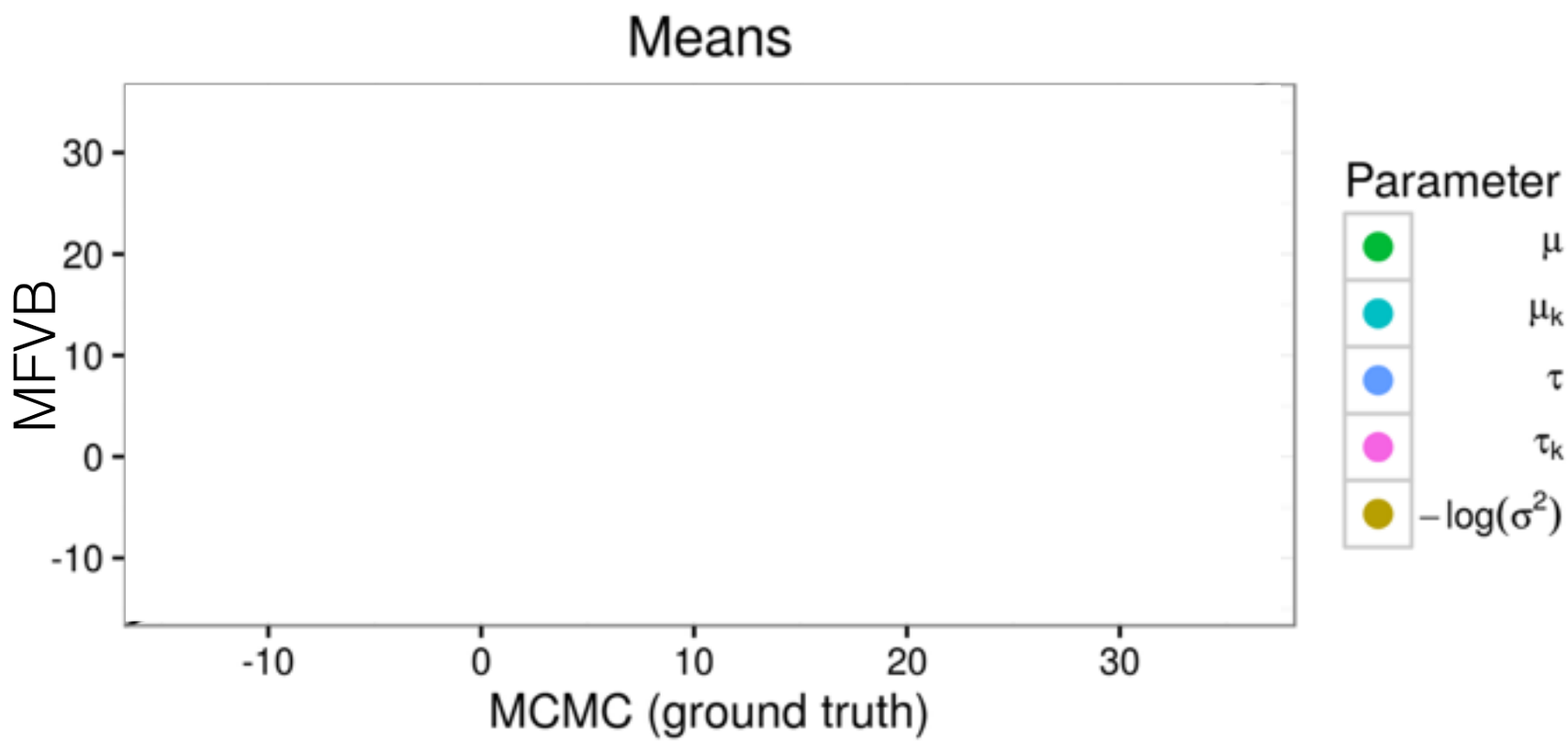

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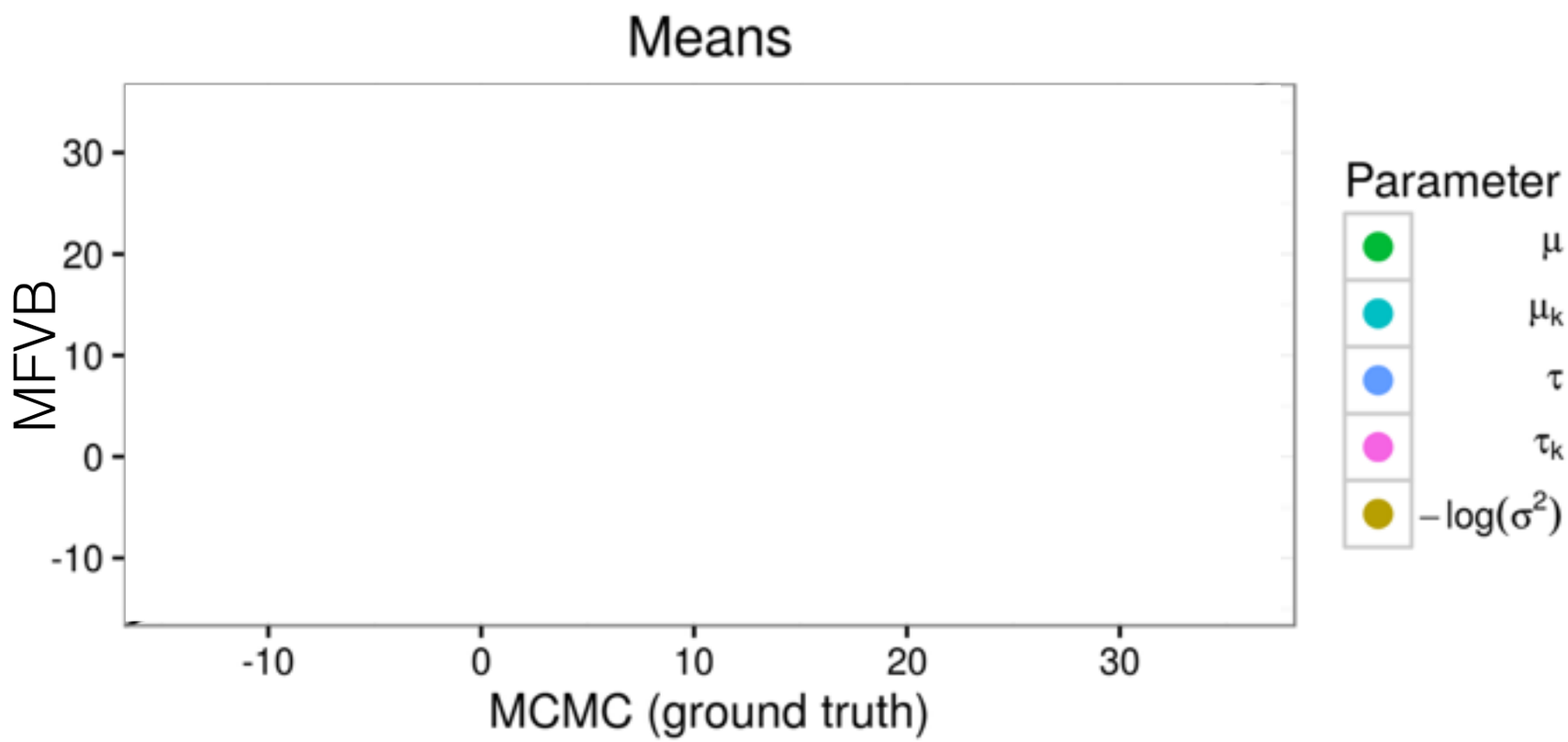

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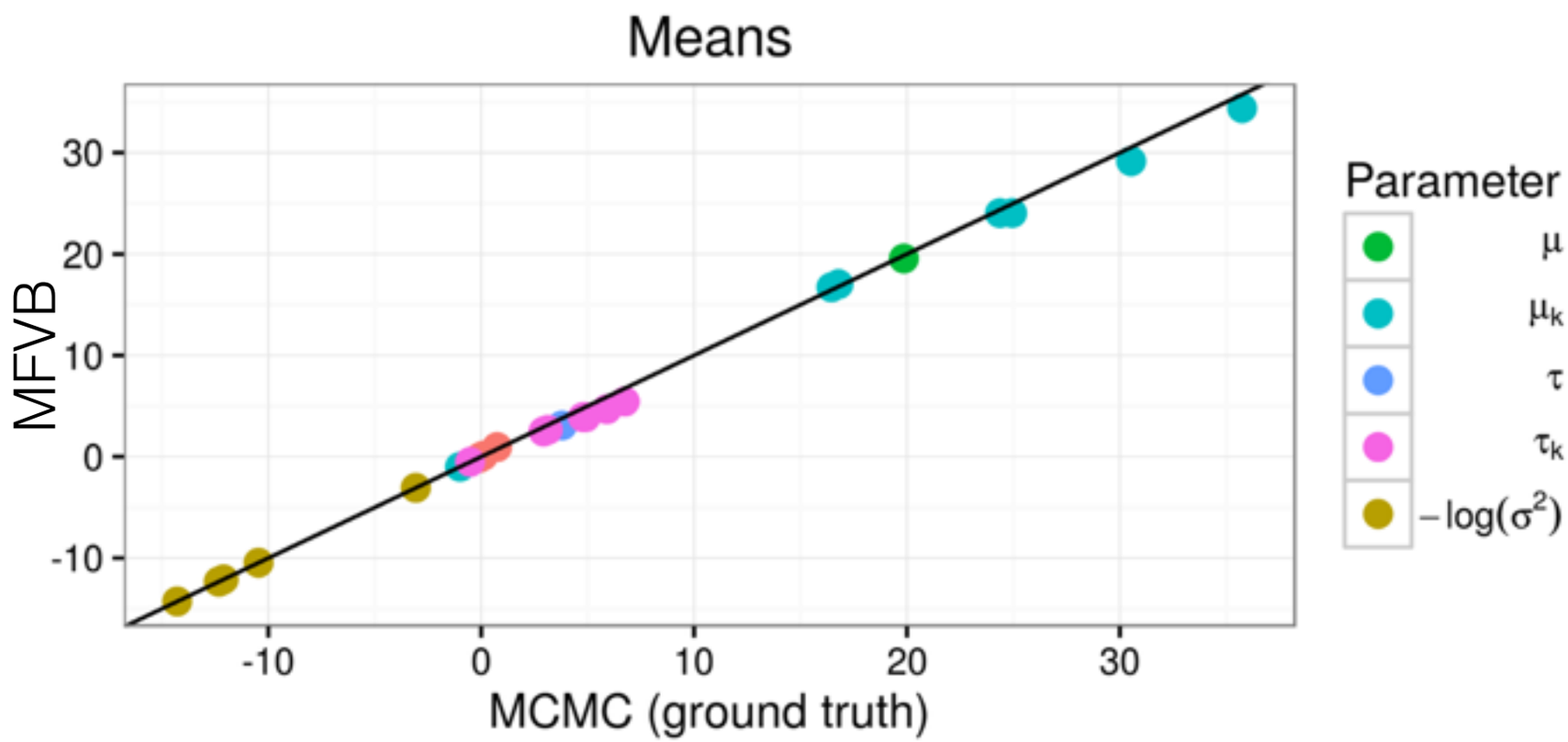

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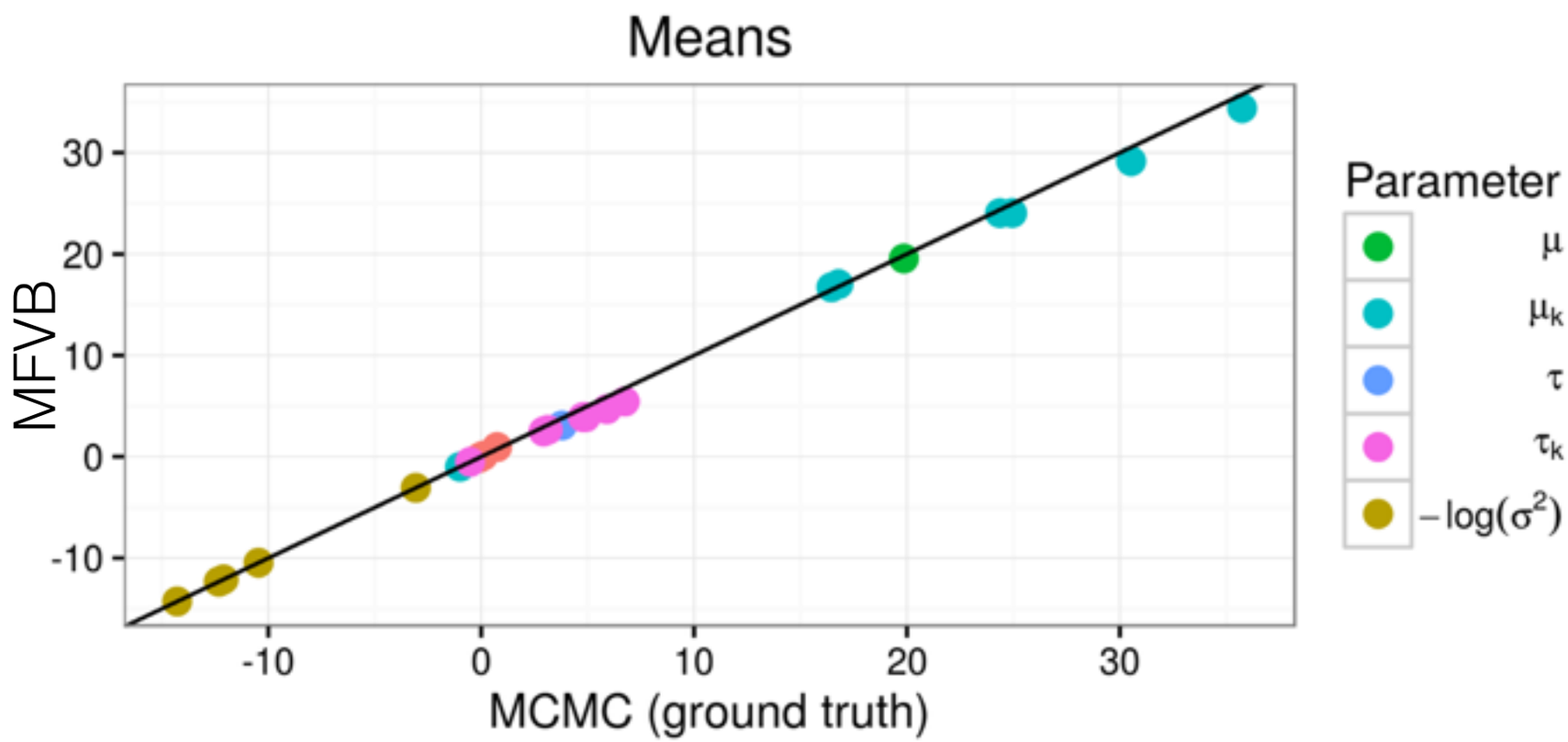

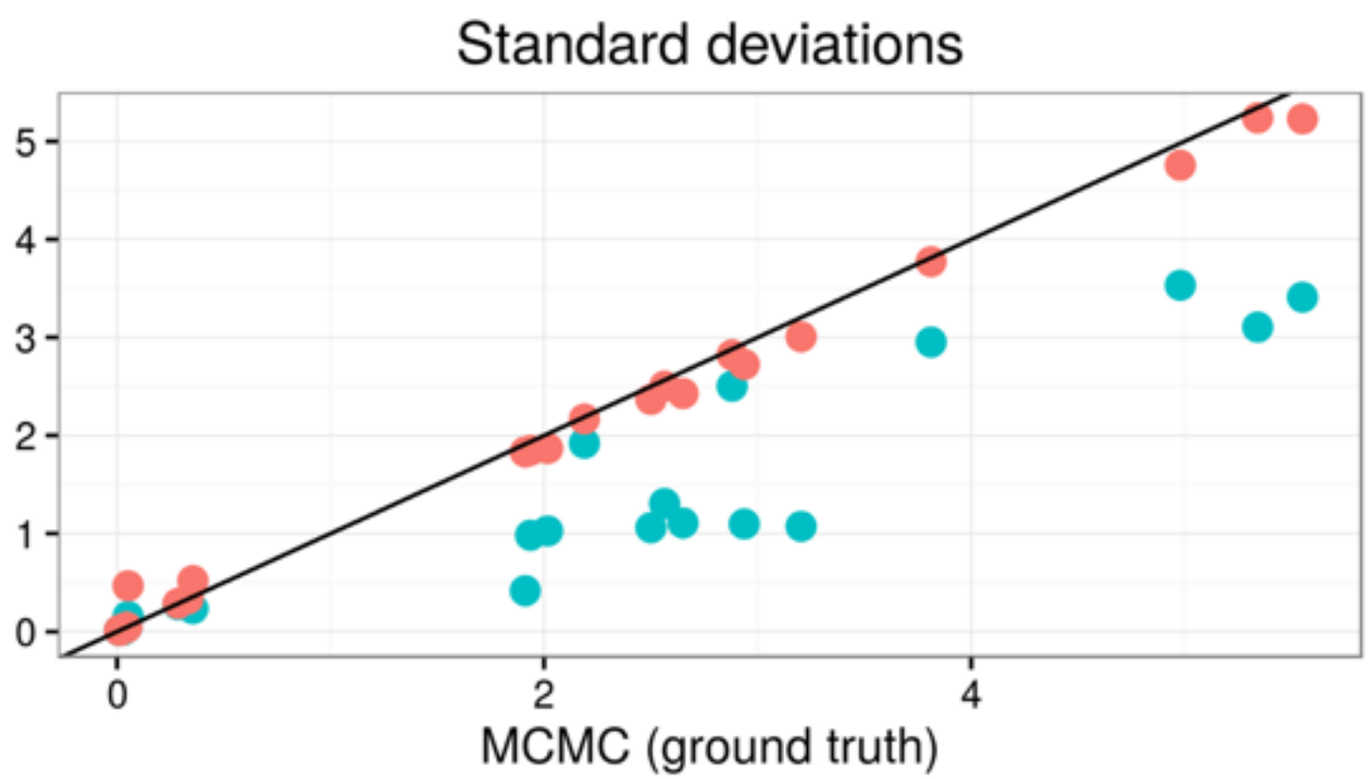

LRVB, MFVB

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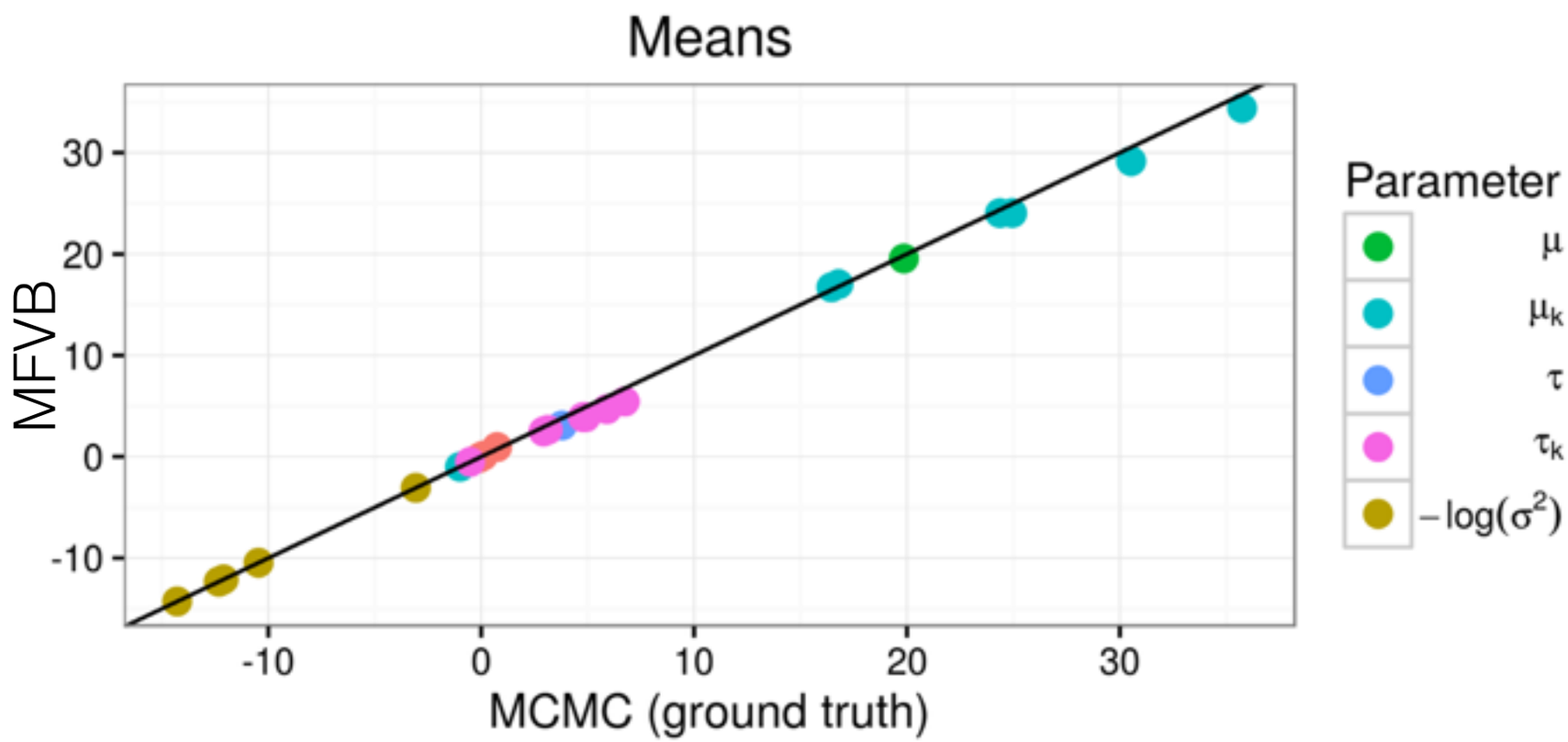

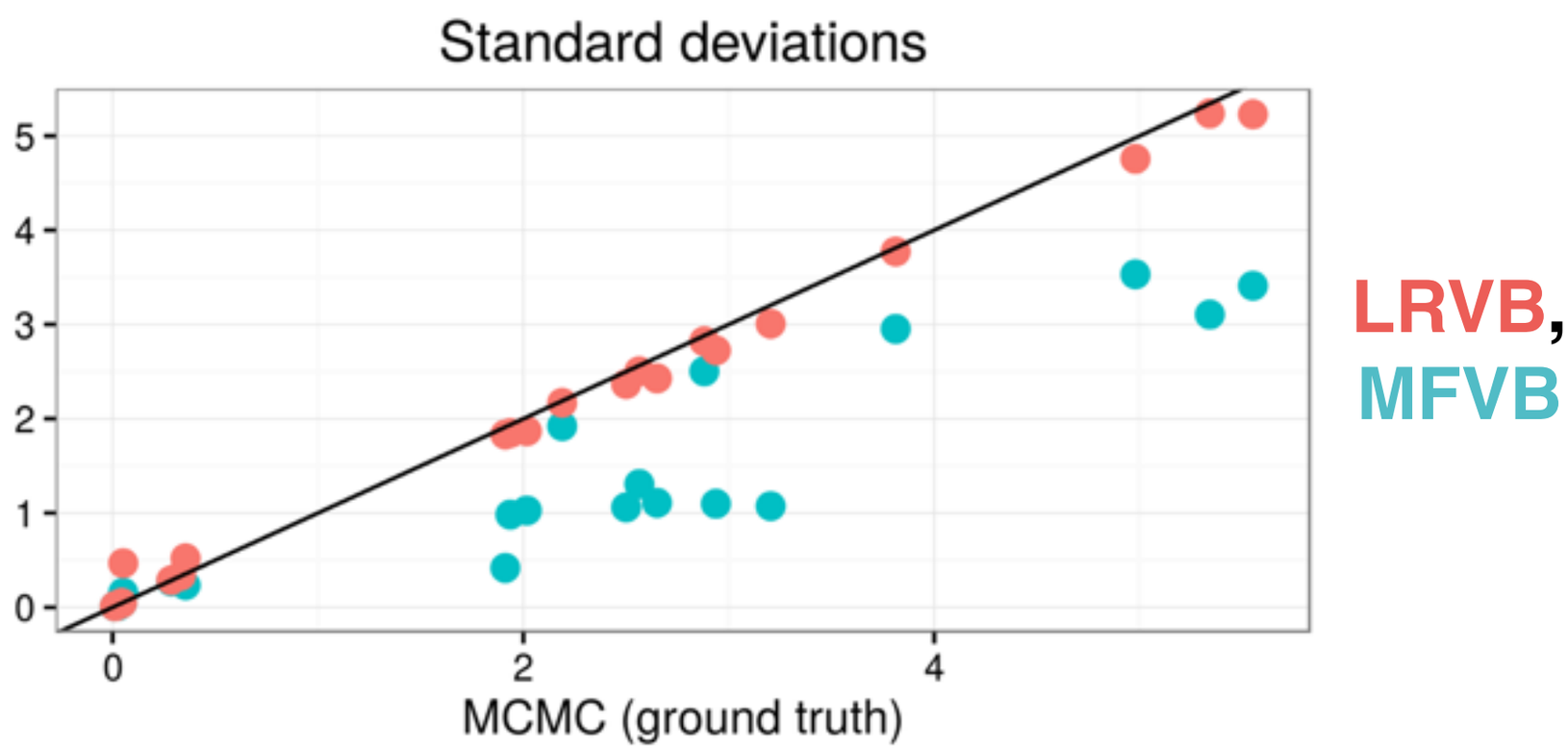

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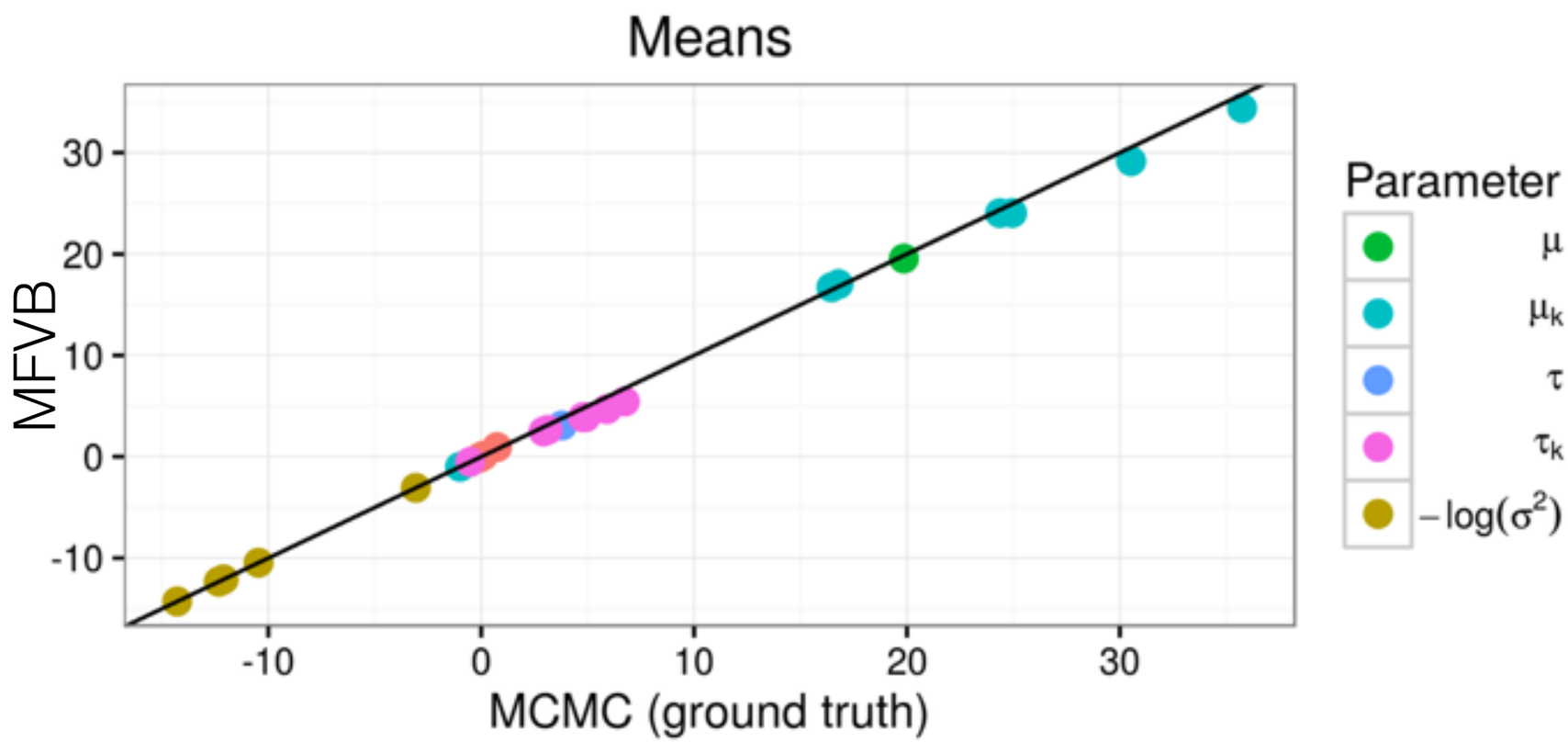

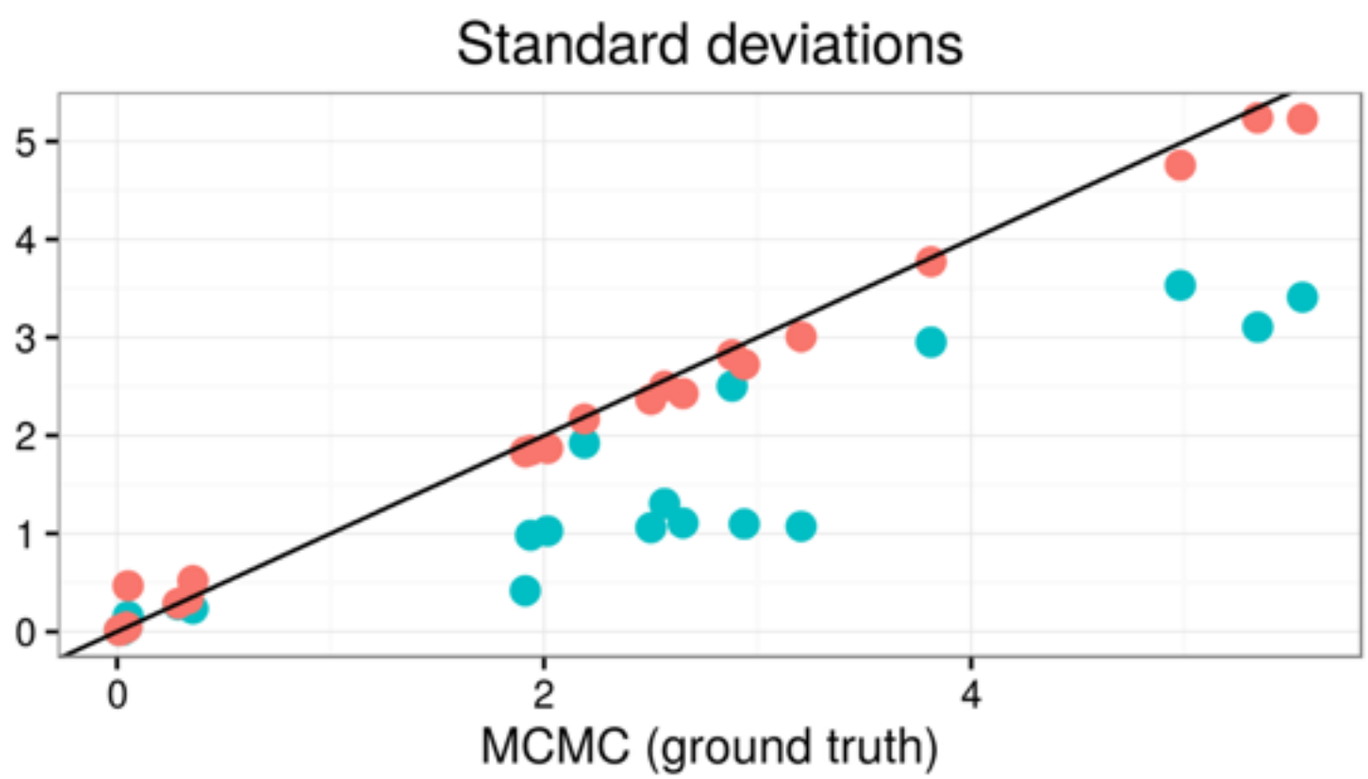

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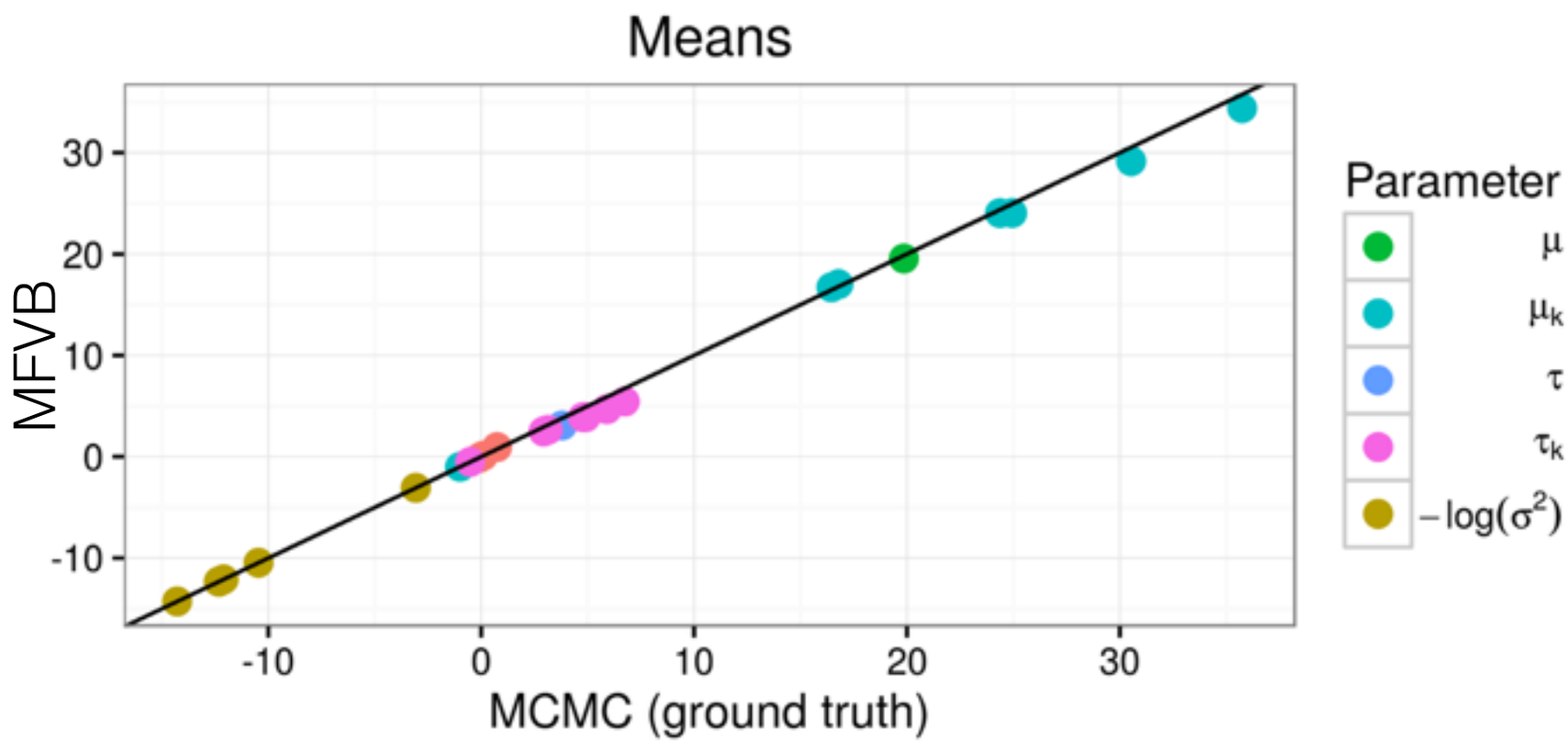

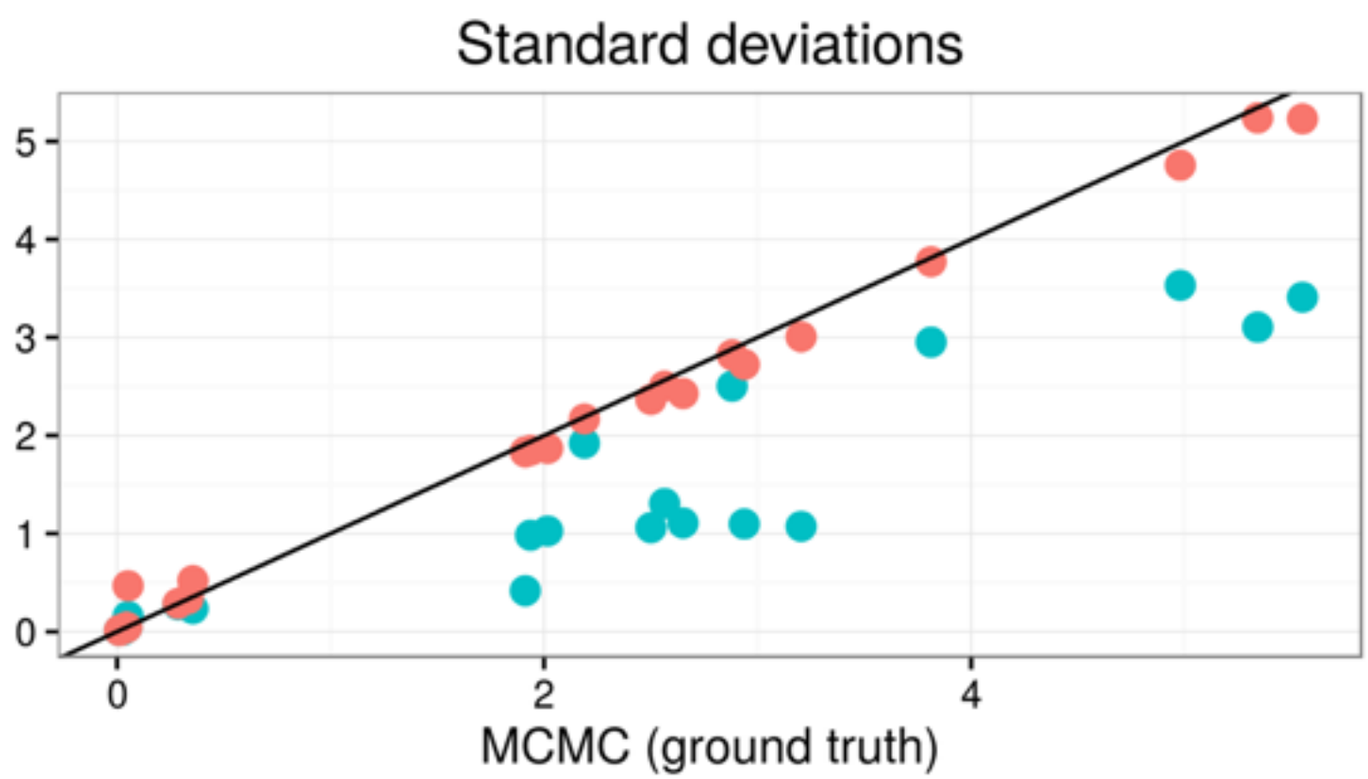

LRVB, MFVB

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- Gaussian mixture model

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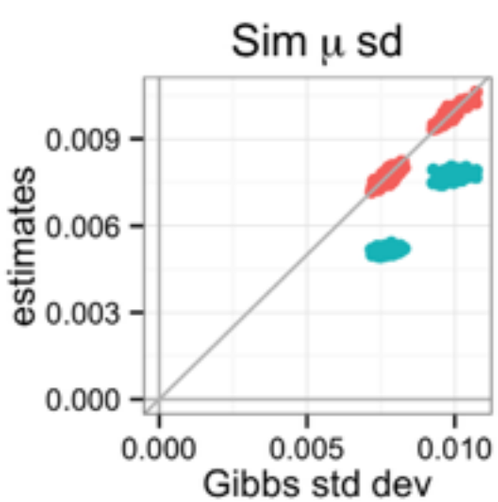

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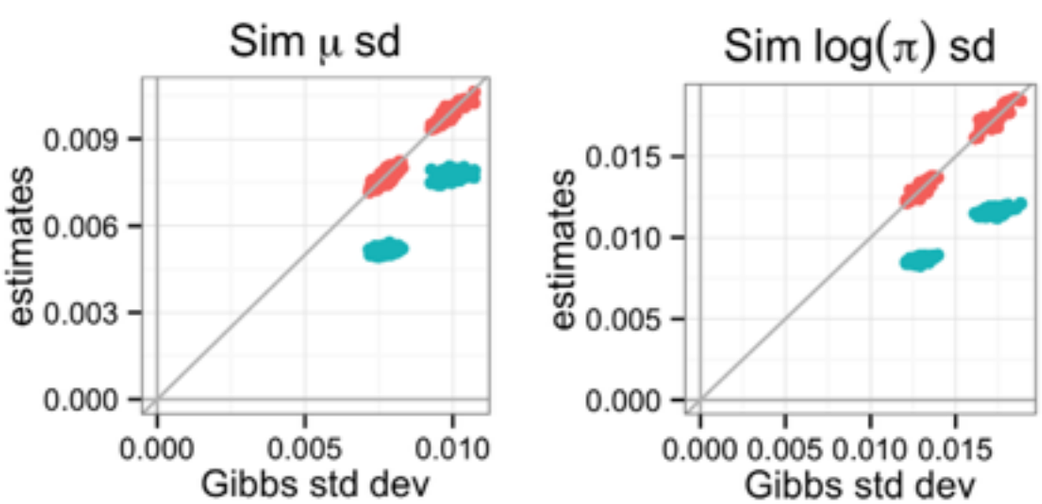

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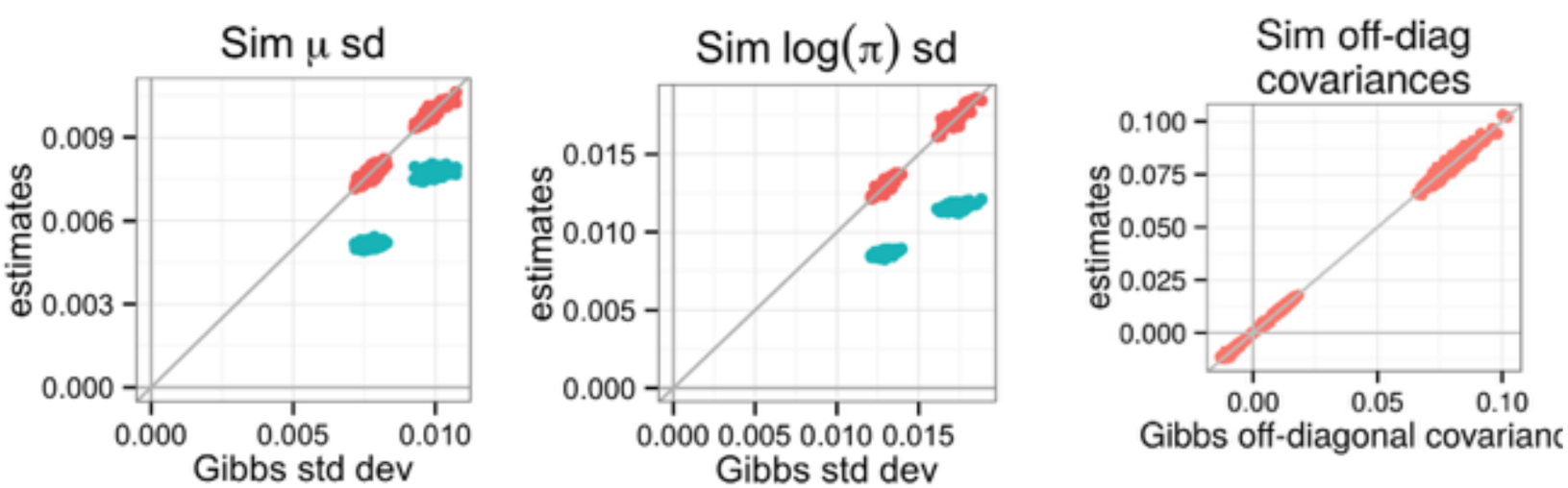

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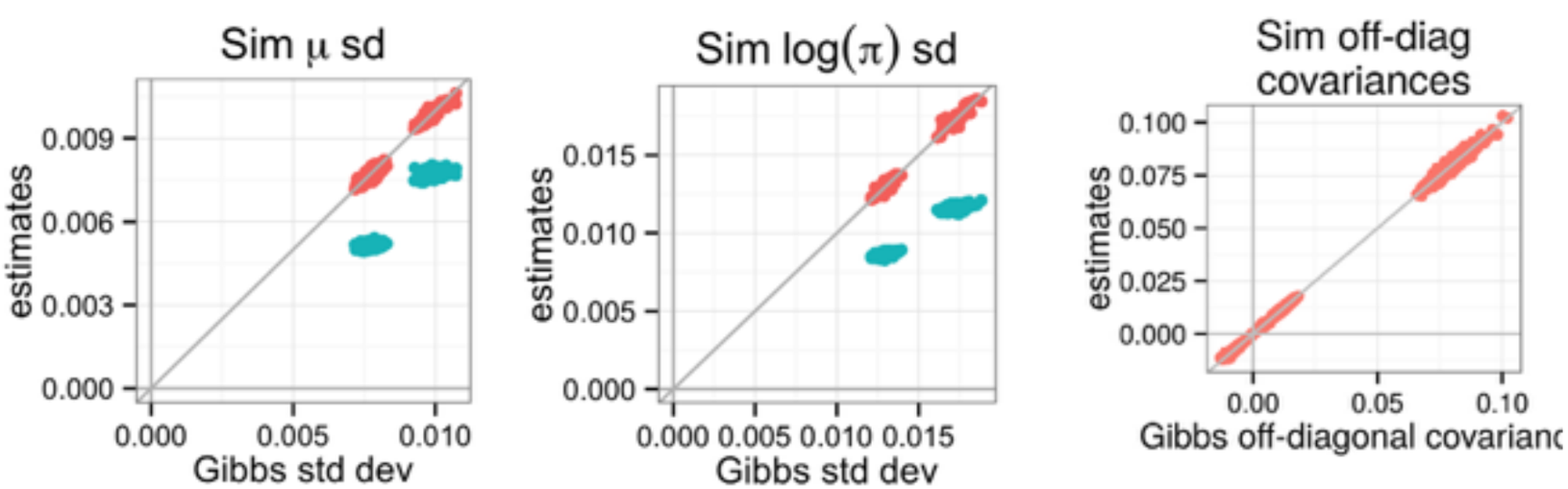

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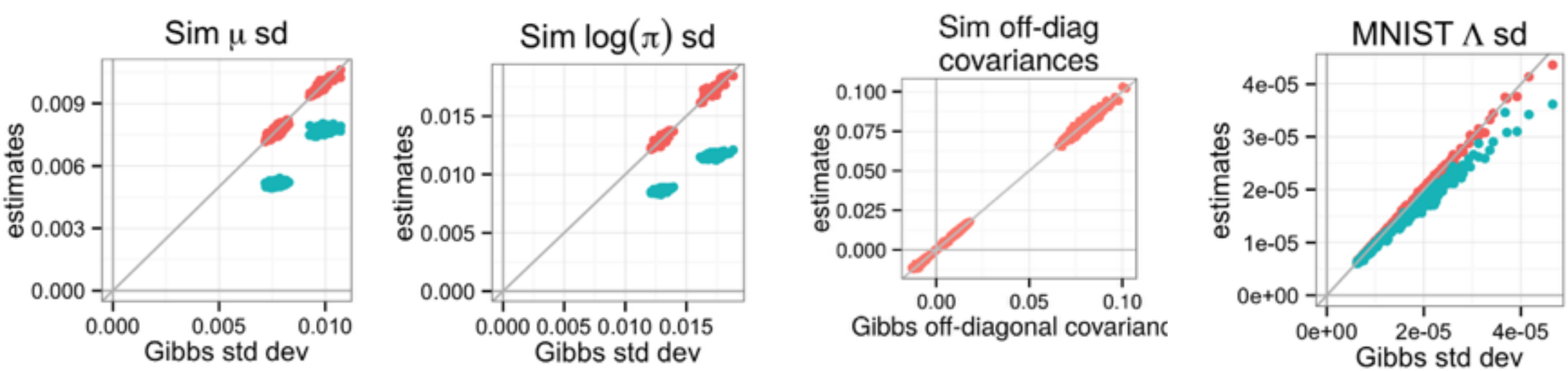

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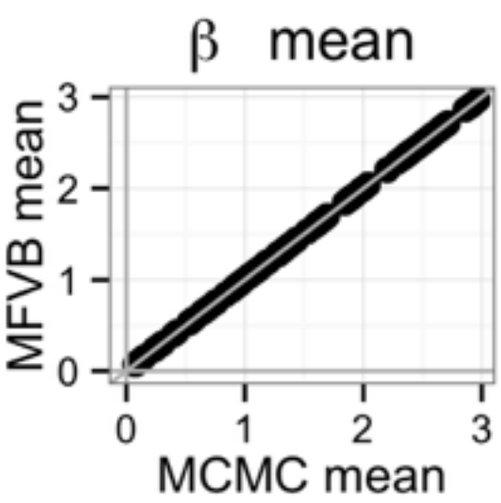

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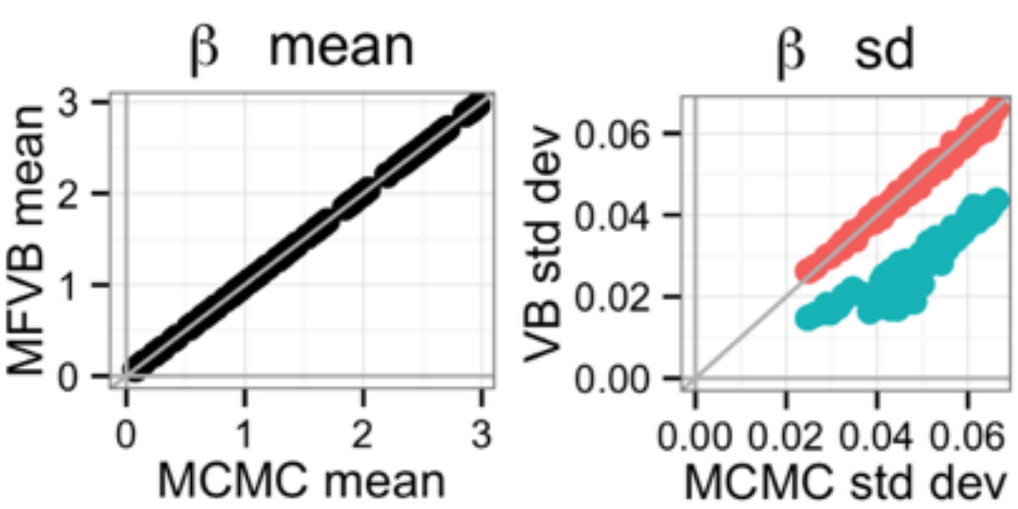

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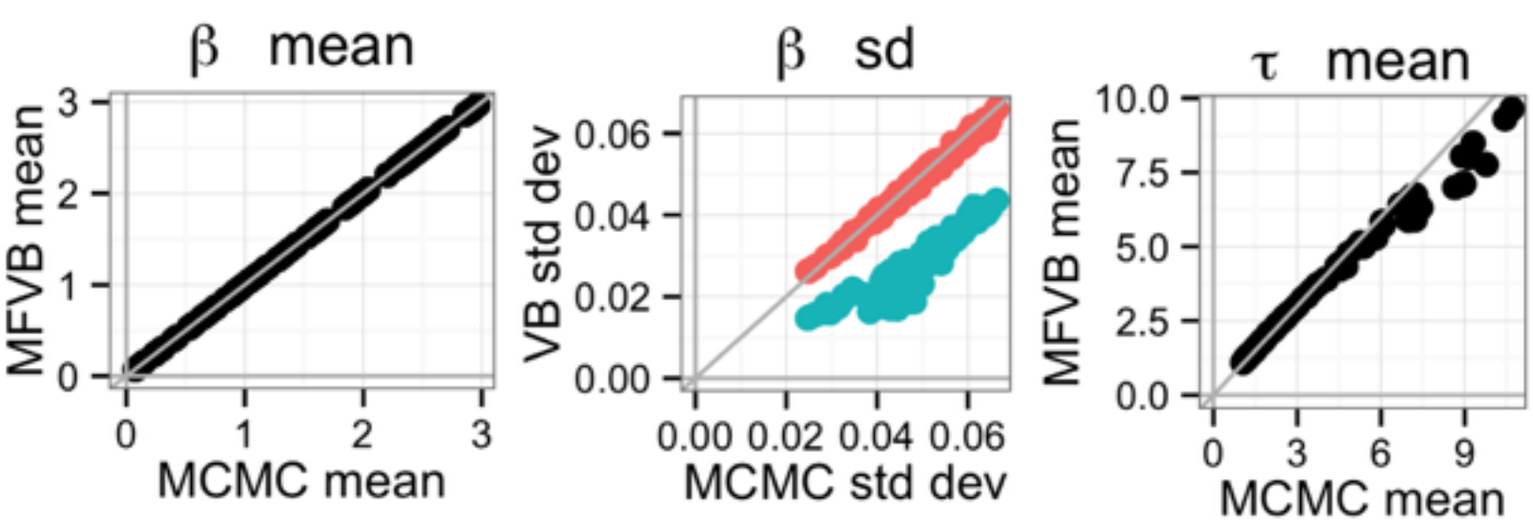

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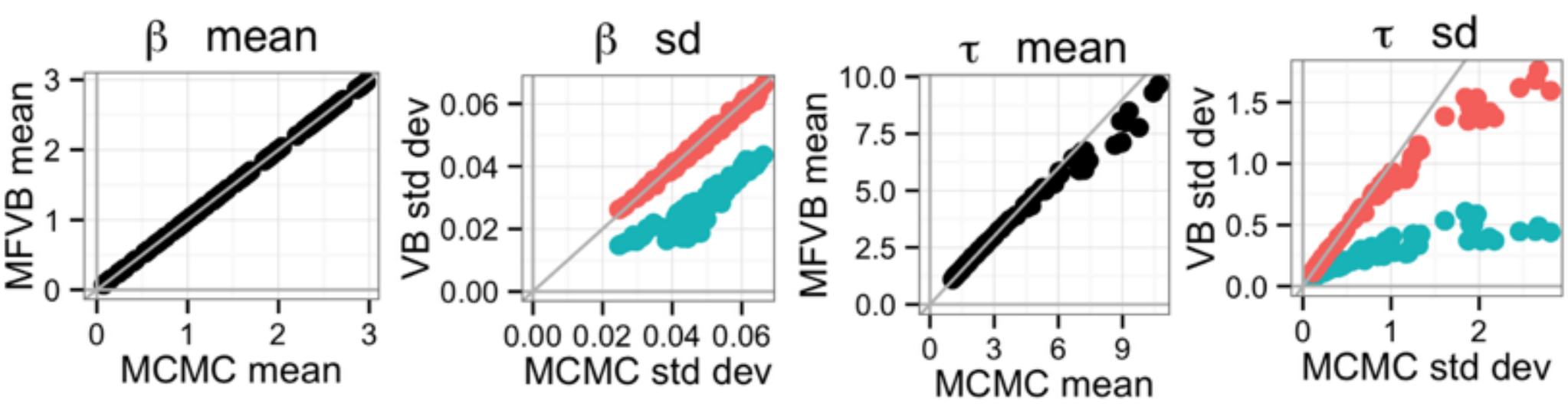

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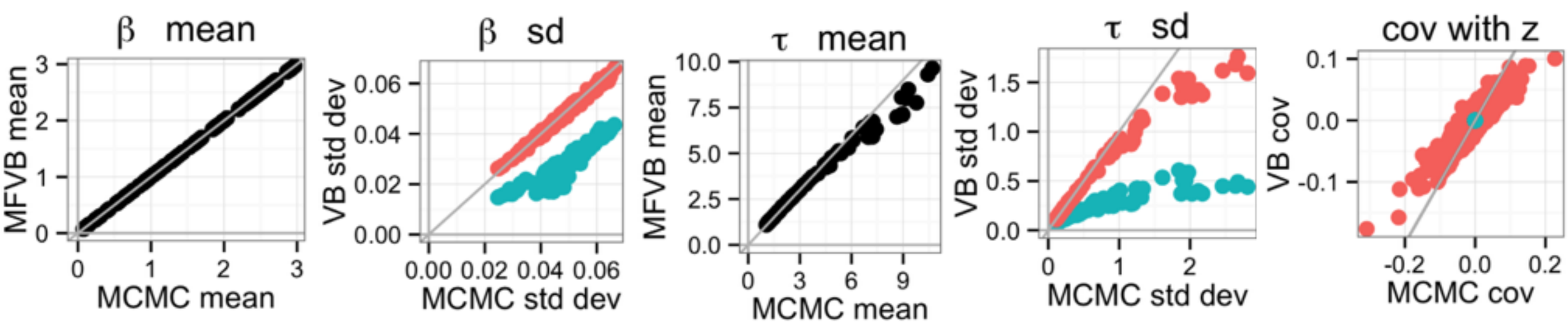

Scaling the matrix inverse

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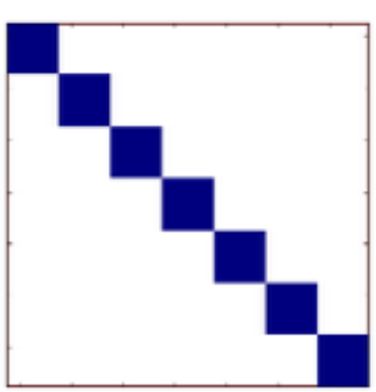
V 

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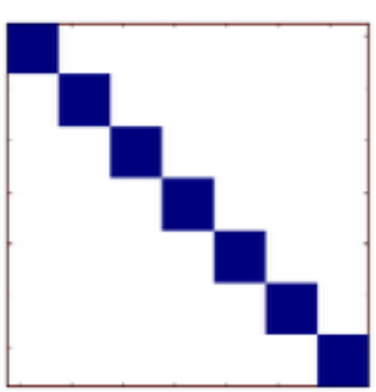
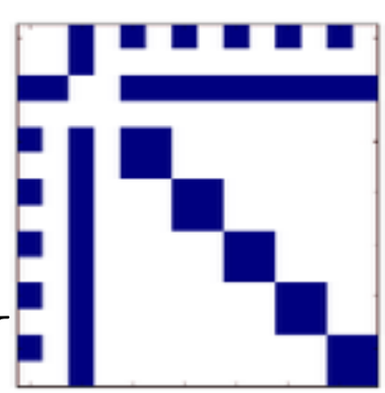
V 

H 

Scaling the matrix inverse

- LRVB estimate $\hat{\Sigma} = (I - VH)^{-1}V$

- Decomposition of parameter vector

$$\theta = (\alpha^T, z^T)^T \qquad H = \begin{bmatrix} H_\alpha & H_{\alpha z} \\ H_{z\alpha} & H_z \end{bmatrix}$$

- Schur complement

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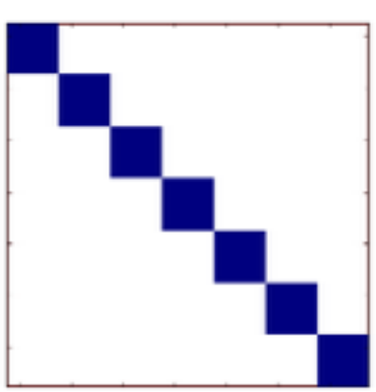
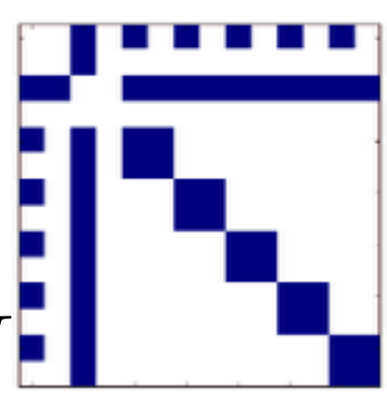
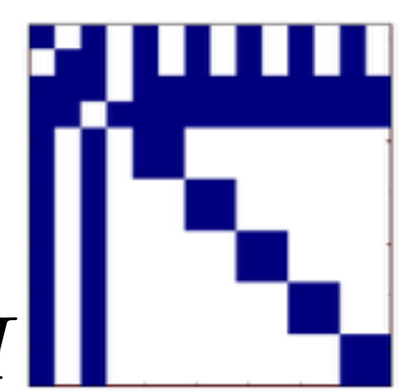
V 

H 

$I - VH$ 

Roadmap

- Variational Bayes as an alternative to MCMC
- Challenges of VB
- Accurate uncertainties from VB
- Accurate robustness quantification from VB

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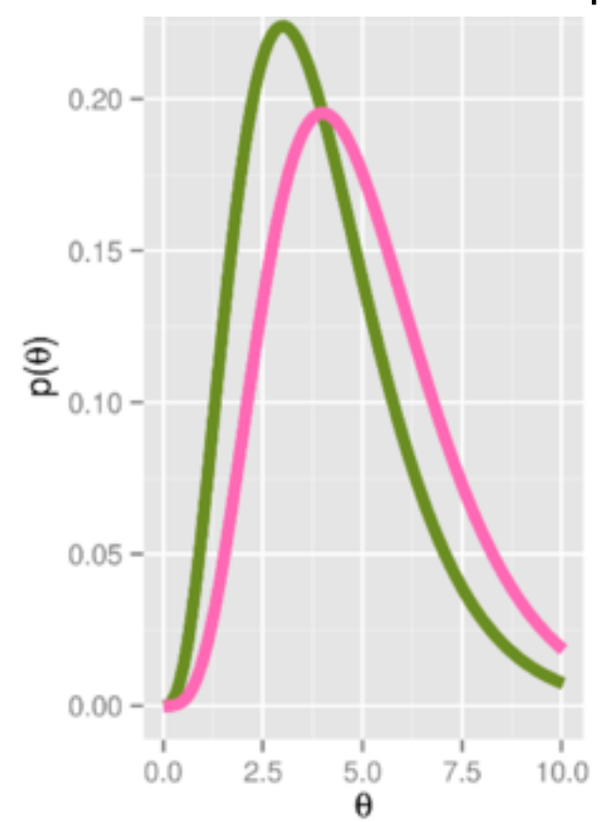

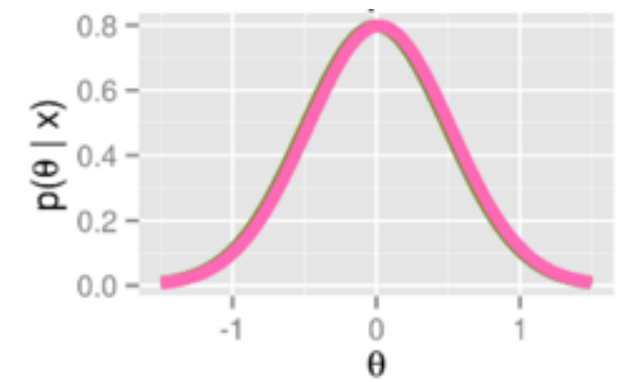

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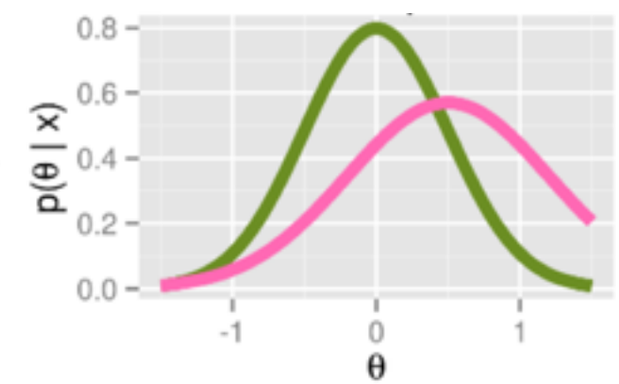

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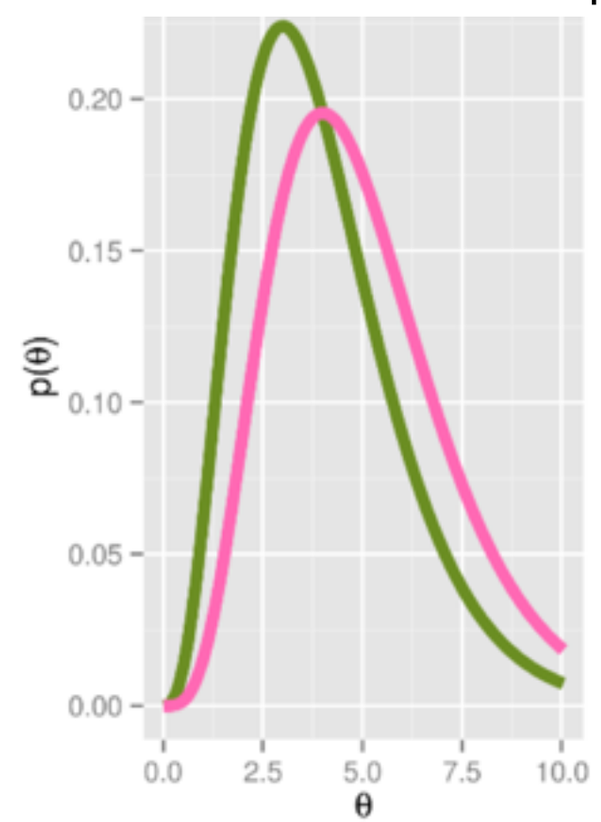

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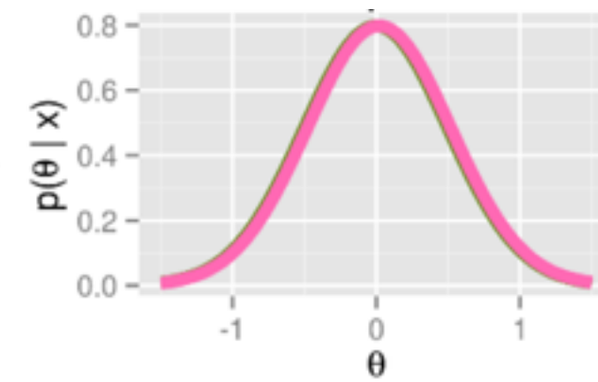

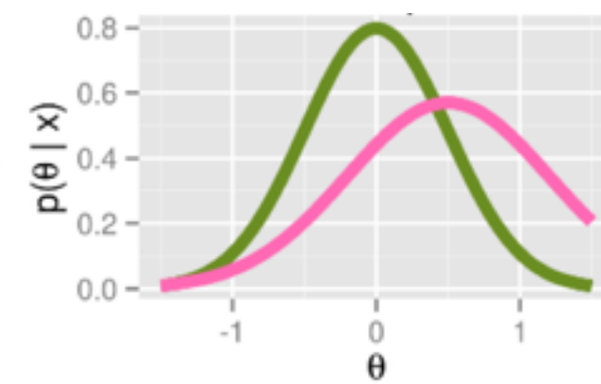

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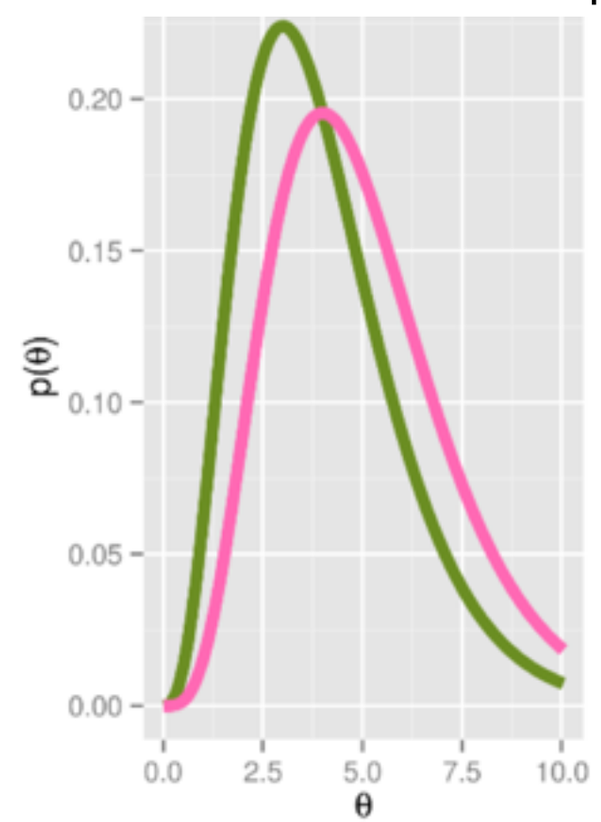

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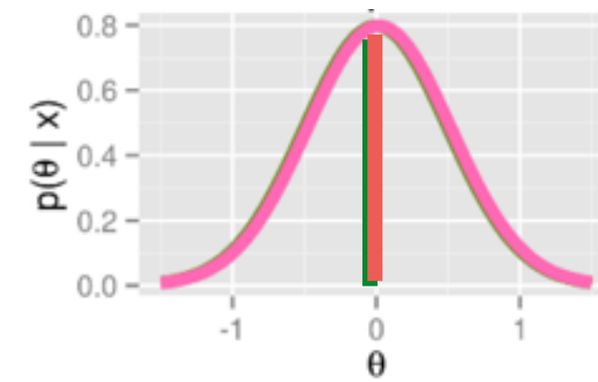

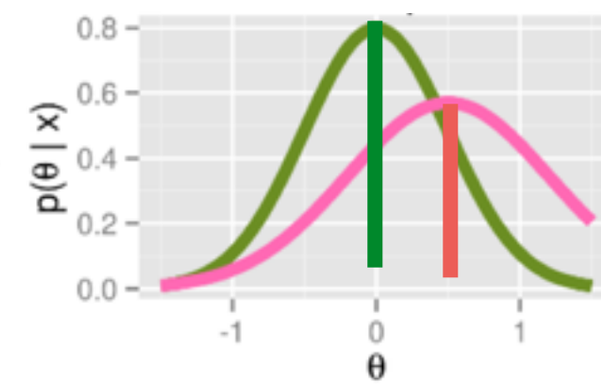

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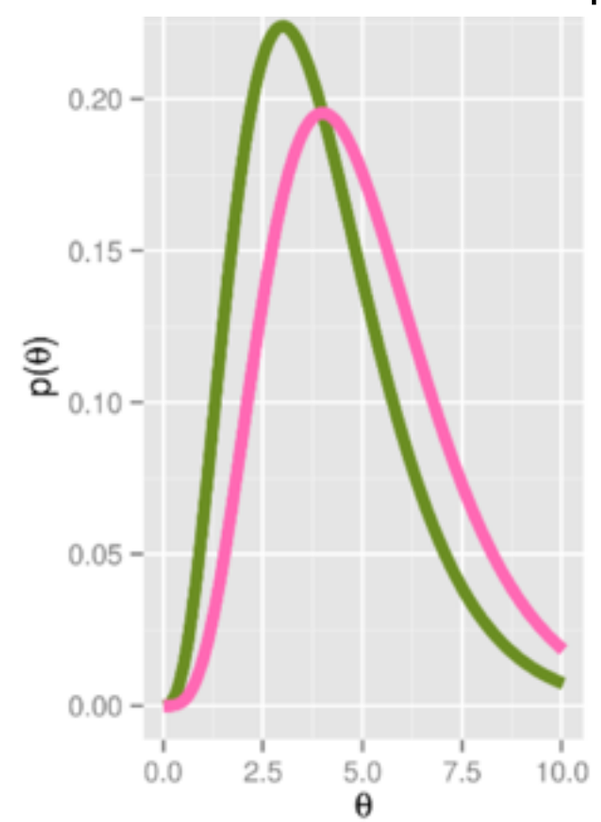

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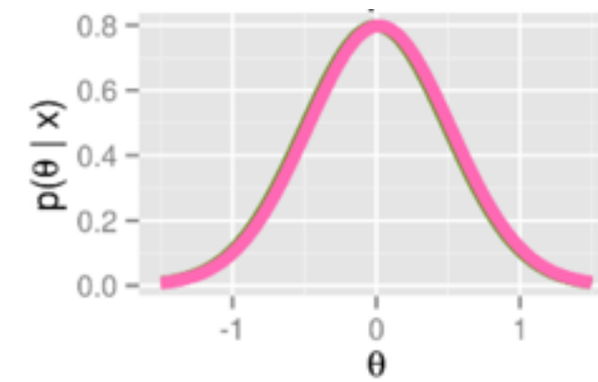

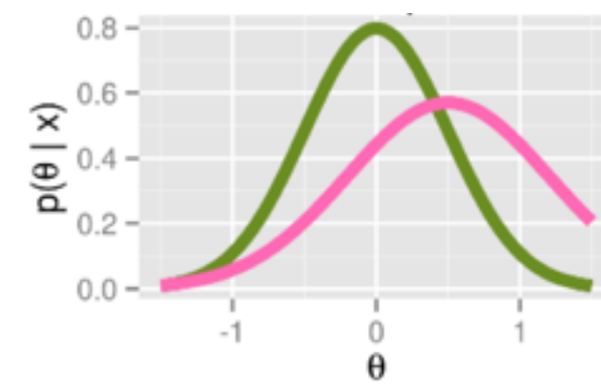

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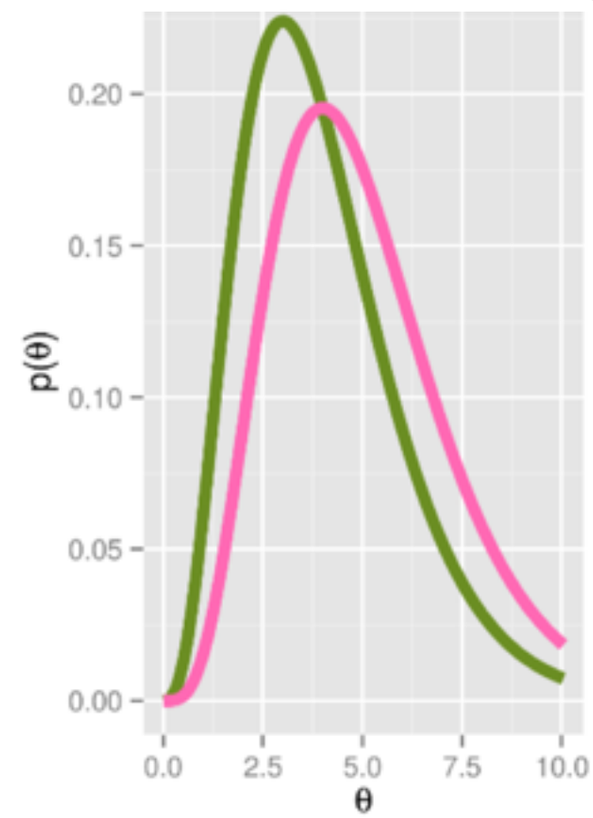

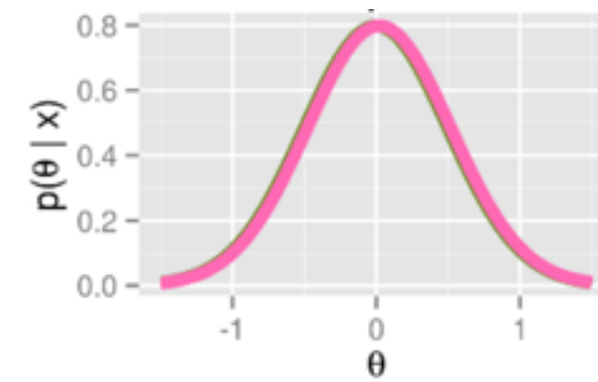

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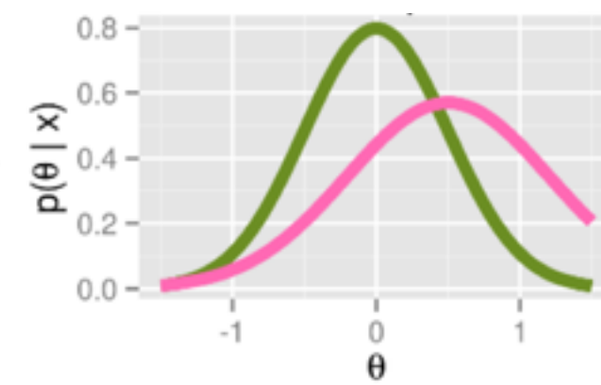

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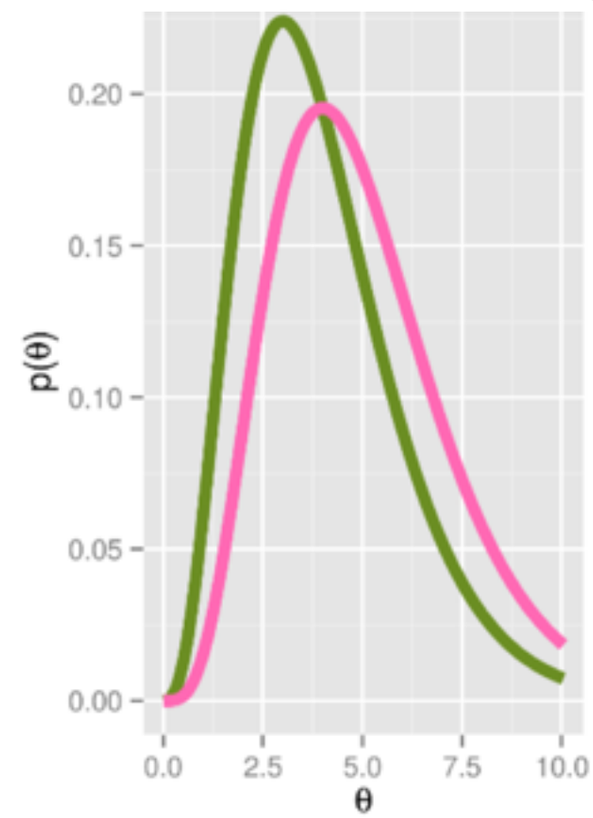

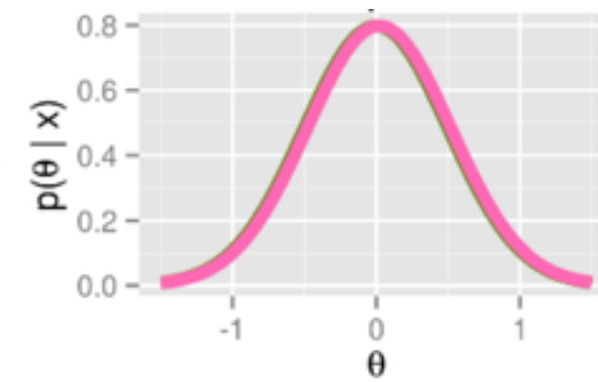

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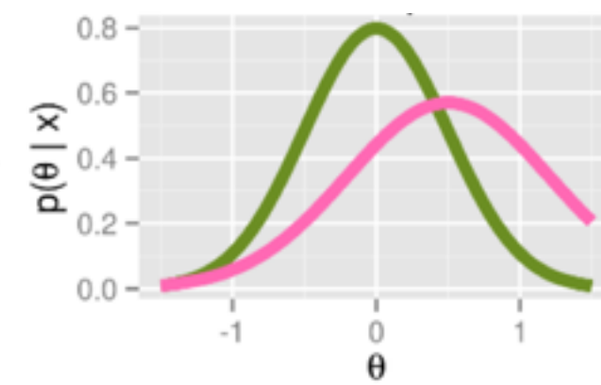

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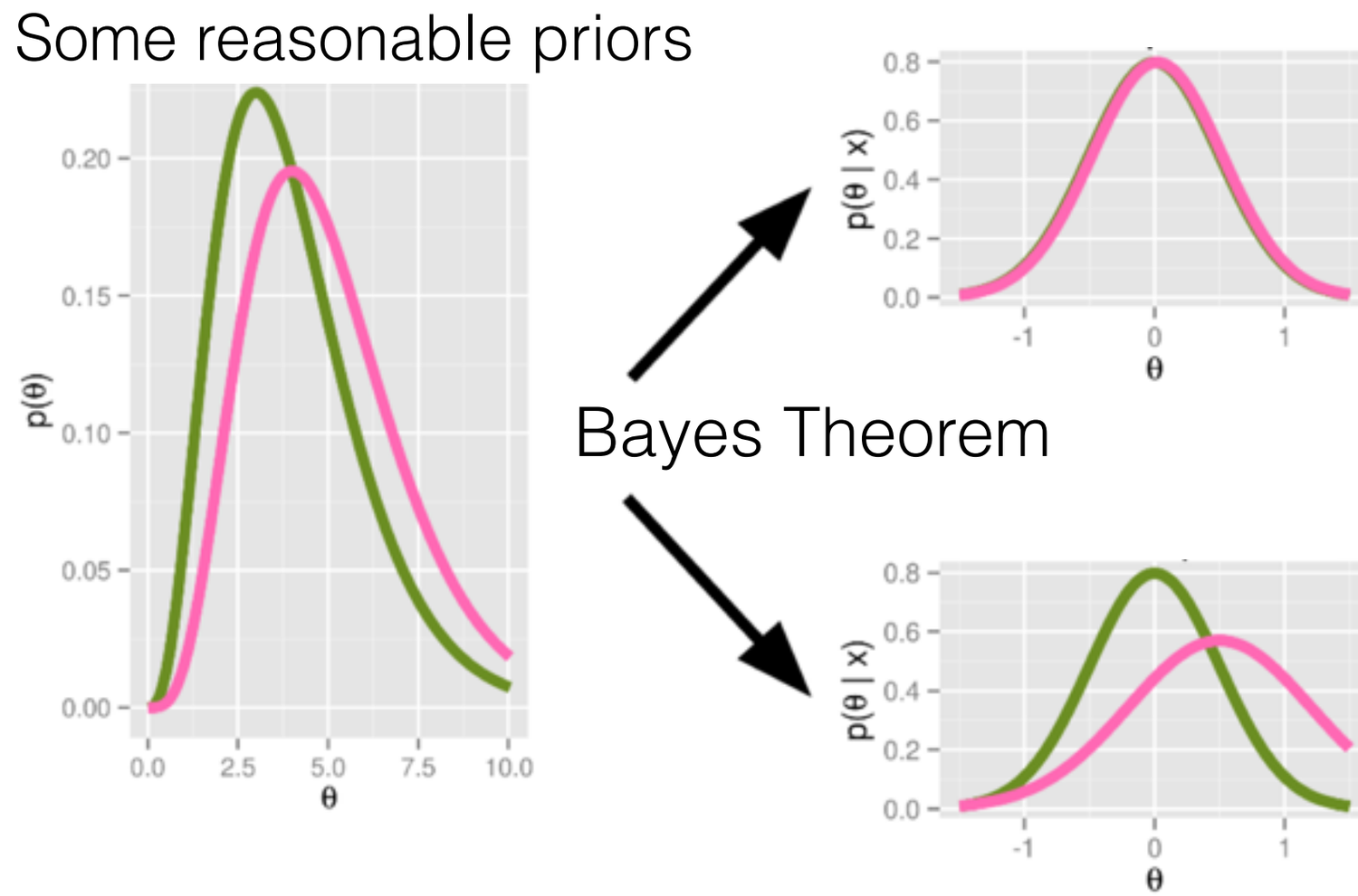

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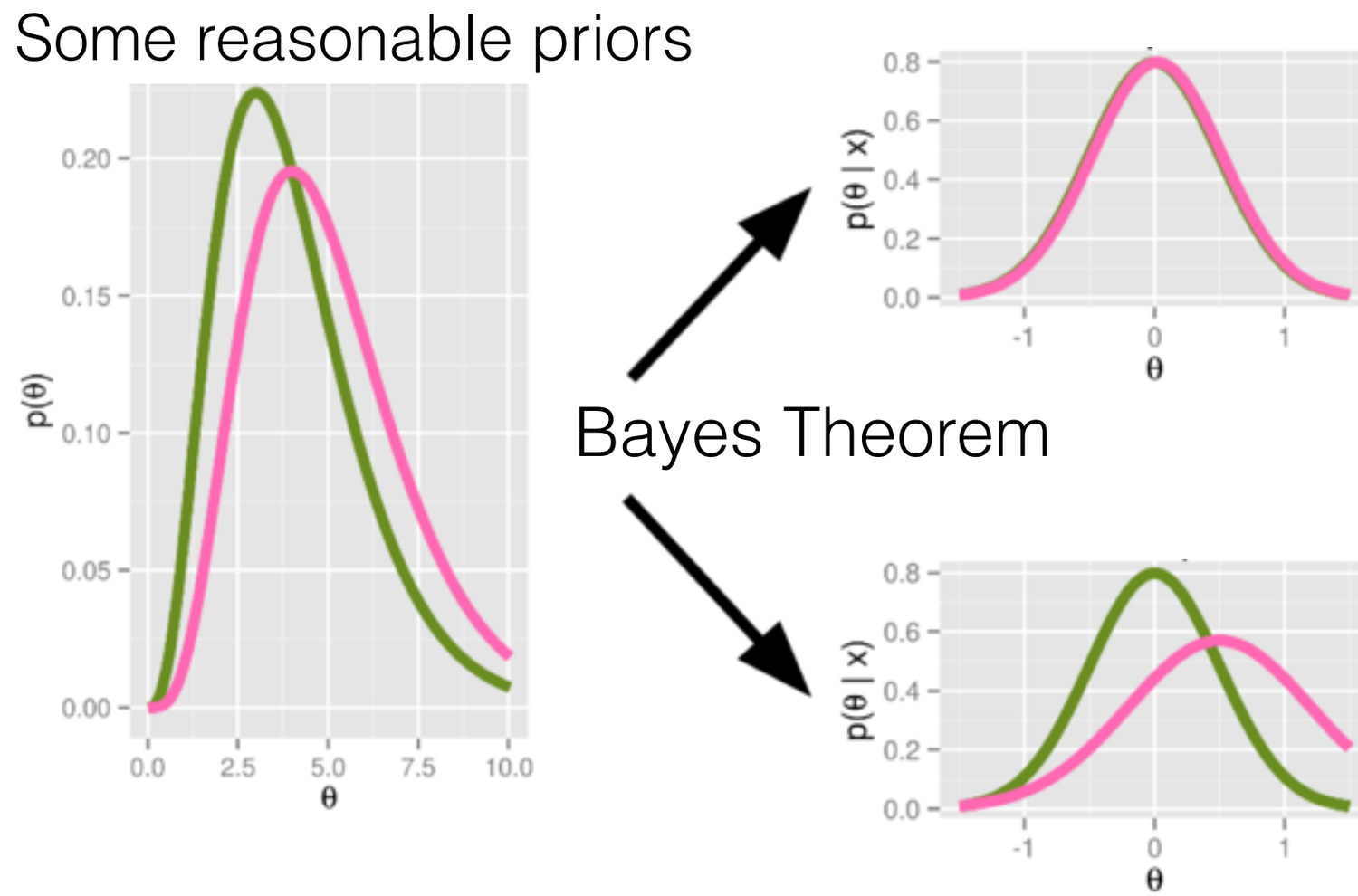

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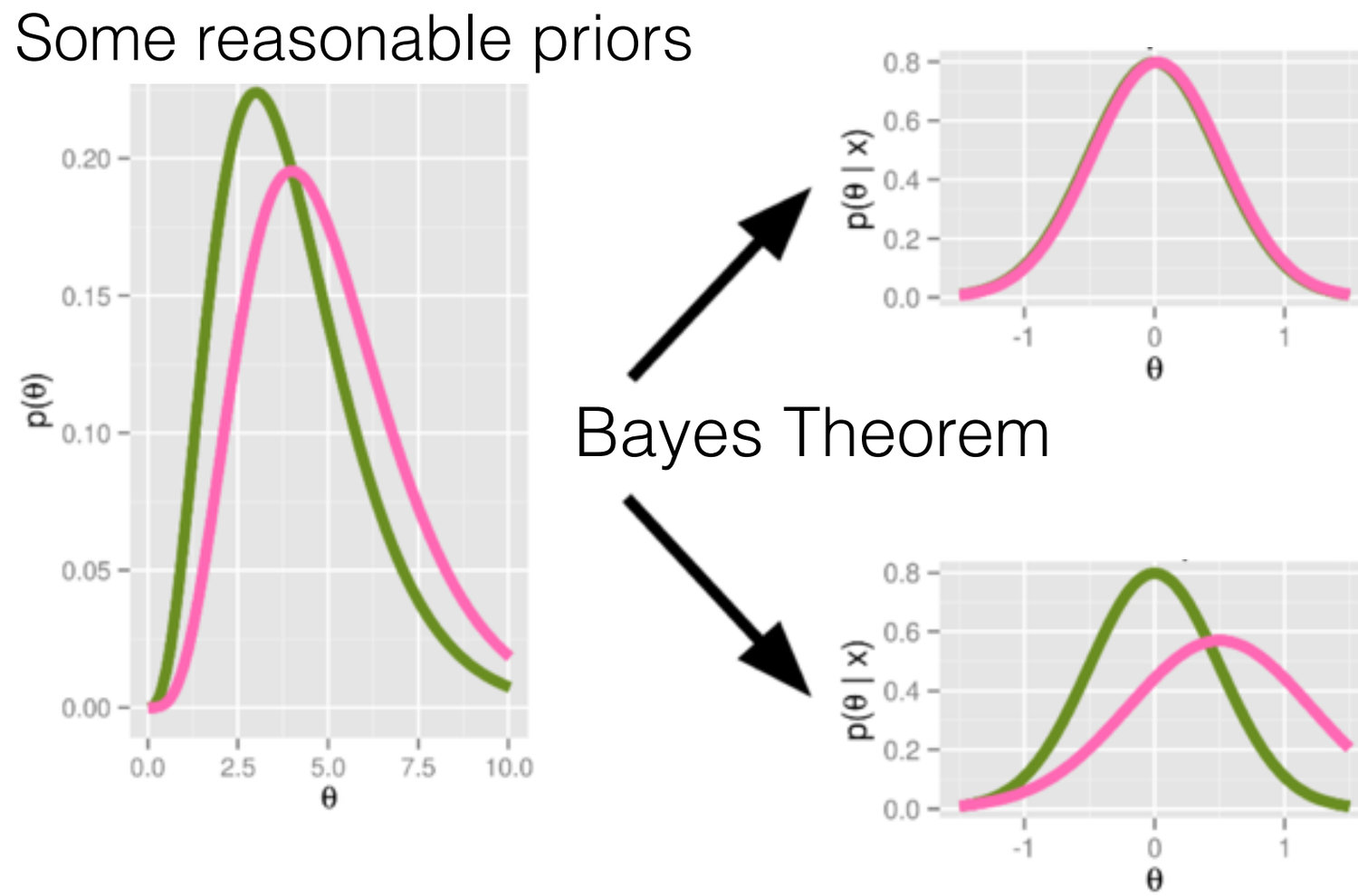

- When q^*_α in exponential family

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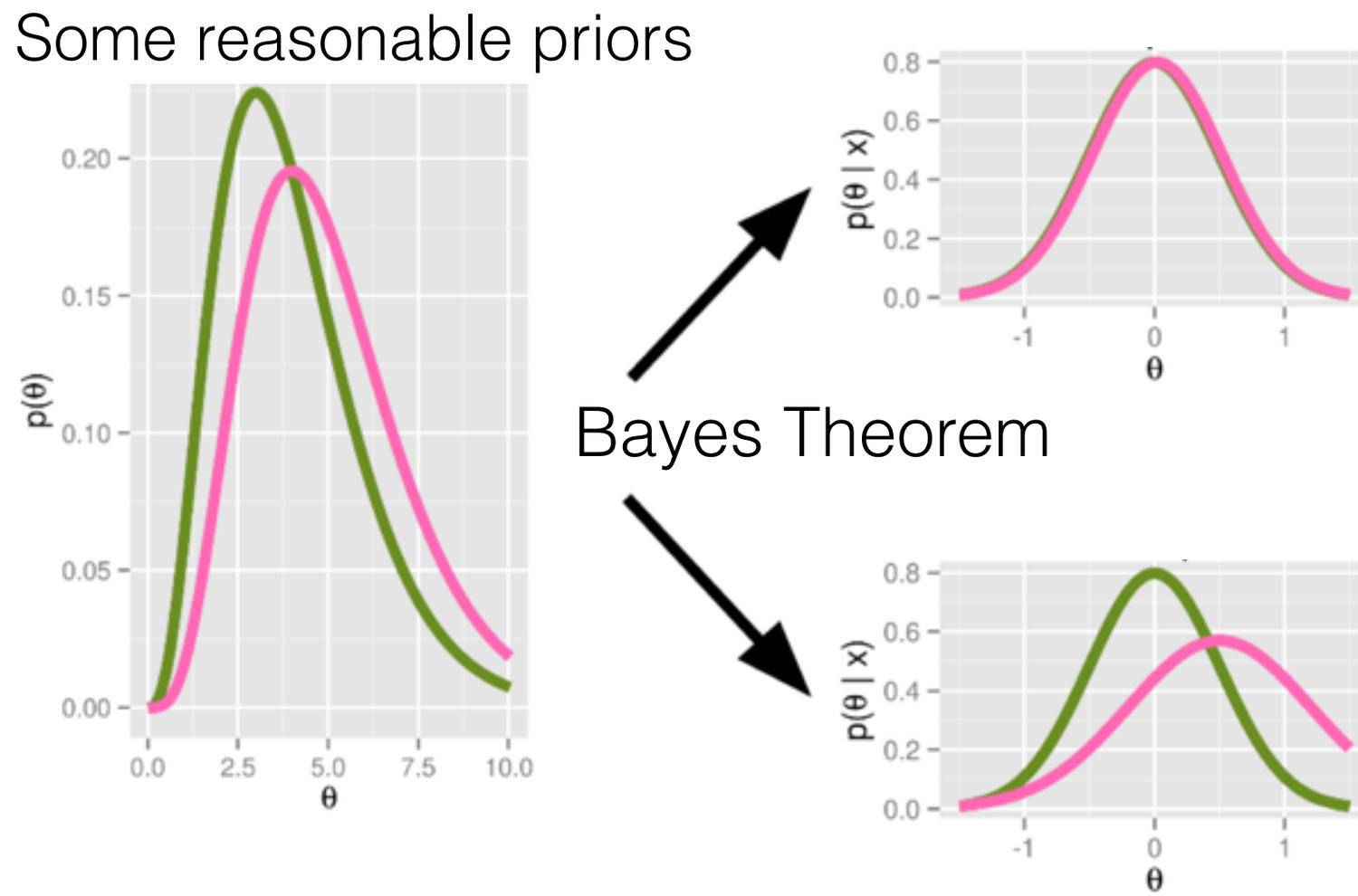

- When q^*_α in exponential family

$$\hat{S} = A\left(\left.\frac{\partial^2 KL}{\partial m \partial m^T}\right|_{m=m^*}\right)^{-1} B$$

Microcredit Experiment

- Simplified from Meager (2015)
- K microcredit trials (Mexico, Mongolia, Bosnia, India, Morocco, Philippines, Ethiopia)
- N_k businesses in kth site (~900 to ~17K)
- Profit of nth business at kth site:

1 if microcredit

profit

$$y_{kn} \overset{indep}{\sim} \mathcal{N}(\mu_k + T_{kn}\tau_k, \sigma_k^2)$$

- Priors and hyperpriors:

$$\begin{pmatrix} \mu_k \\ \tau_k \end{pmatrix} \overset{iid}{\sim} \mathcal{N}\left(\begin{pmatrix} \mu \\ \tau \end{pmatrix}, C\right) \qquad \begin{pmatrix} \mu \\ \tau \end{pmatrix} \overset{iid}{\sim} \mathcal{N}\left(\begin{pmatrix} \mu_0 \\ \tau_0 \end{pmatrix}, \Lambda^{-1}\right)$$

$$\sigma_k^{-2} \overset{iid}{\sim} \Gamma(a, b) \qquad C \sim \text{Sep\&LKJ}(\eta, c, d)$$

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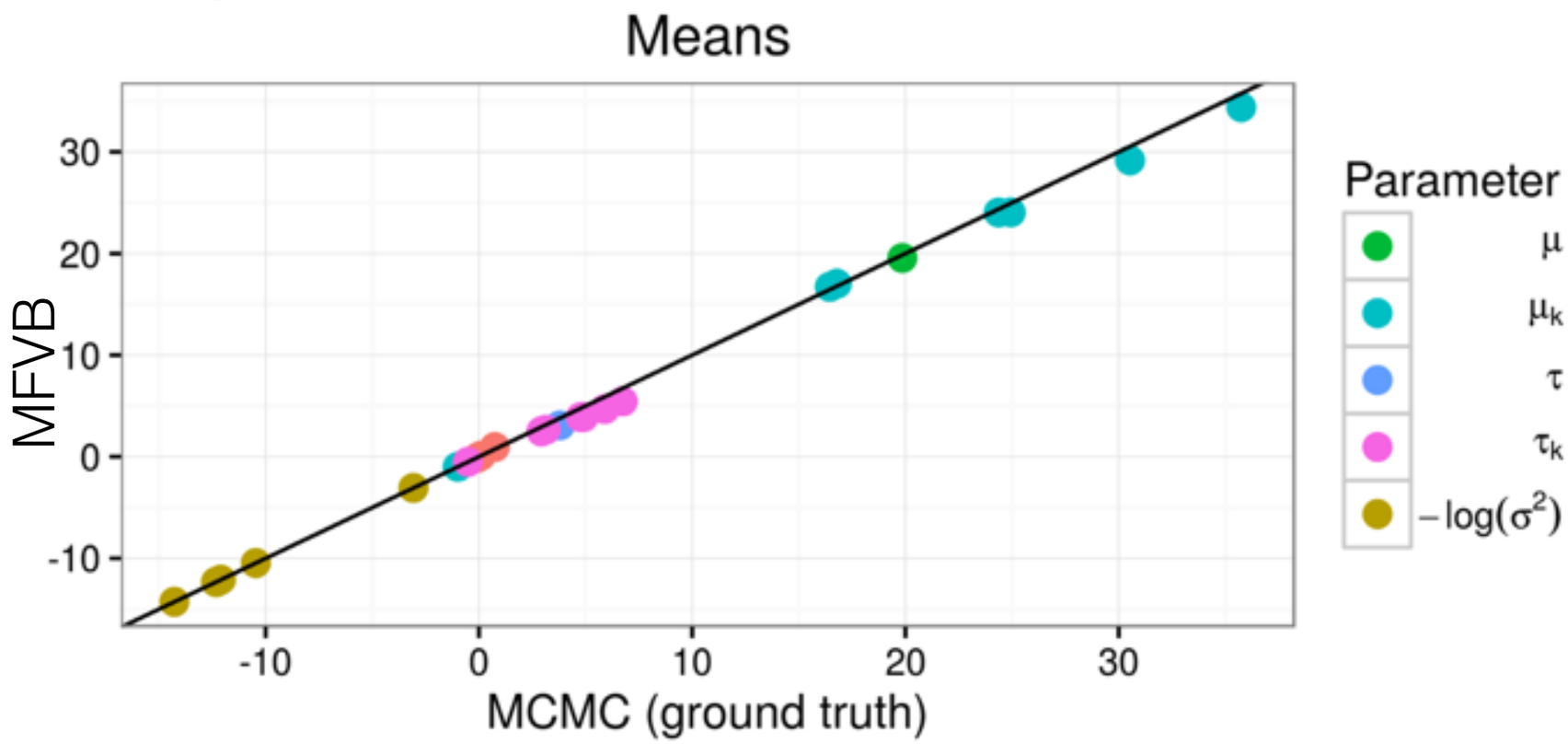

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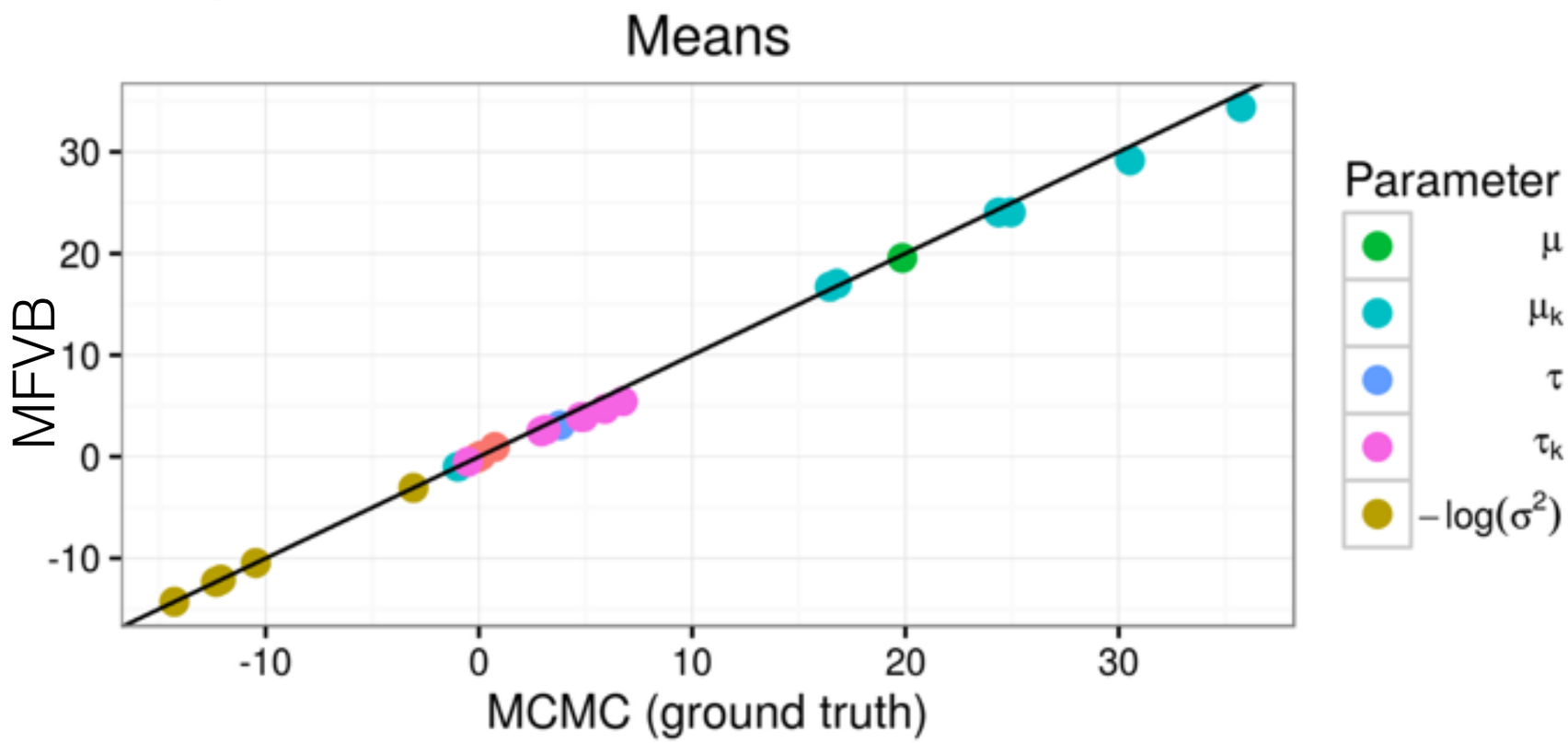

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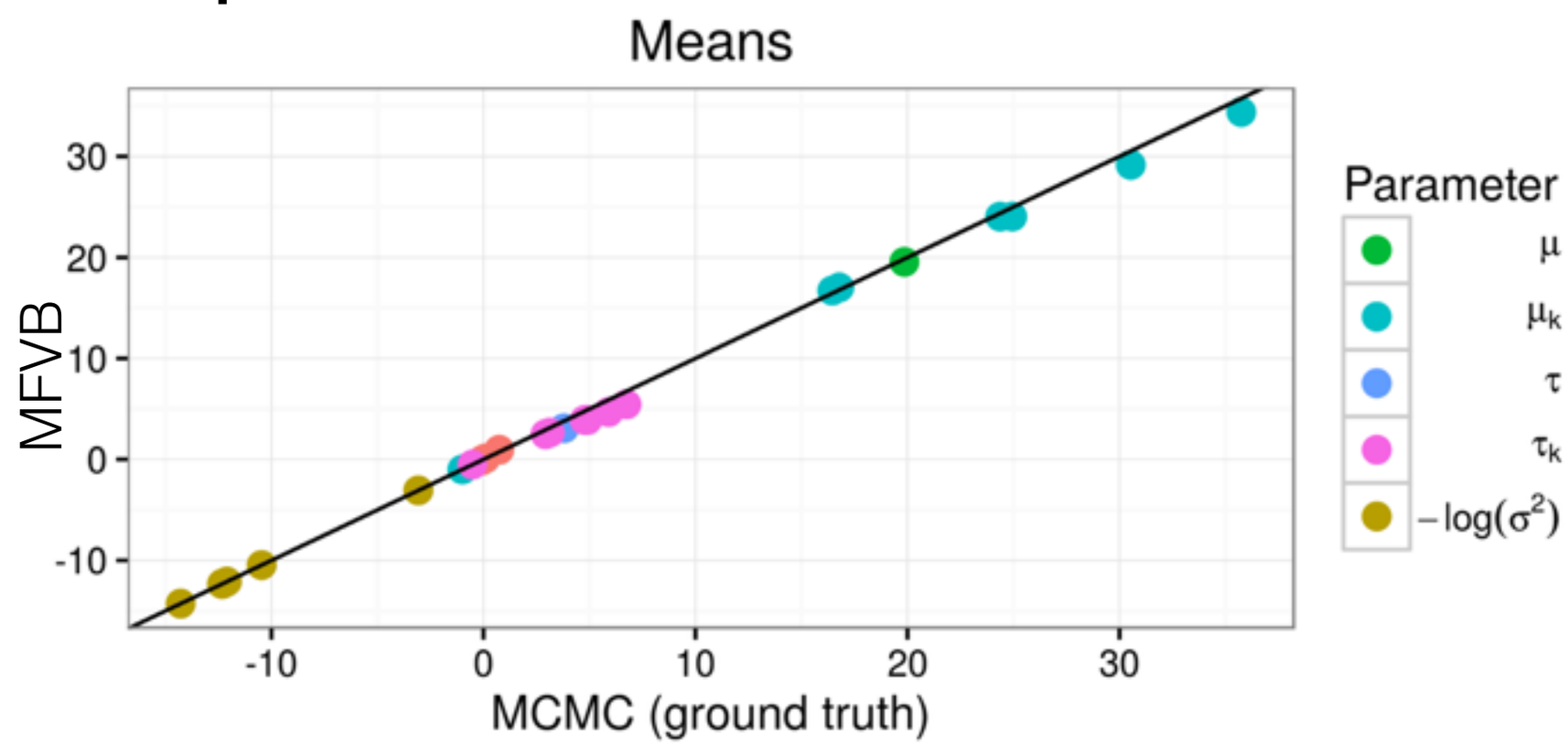

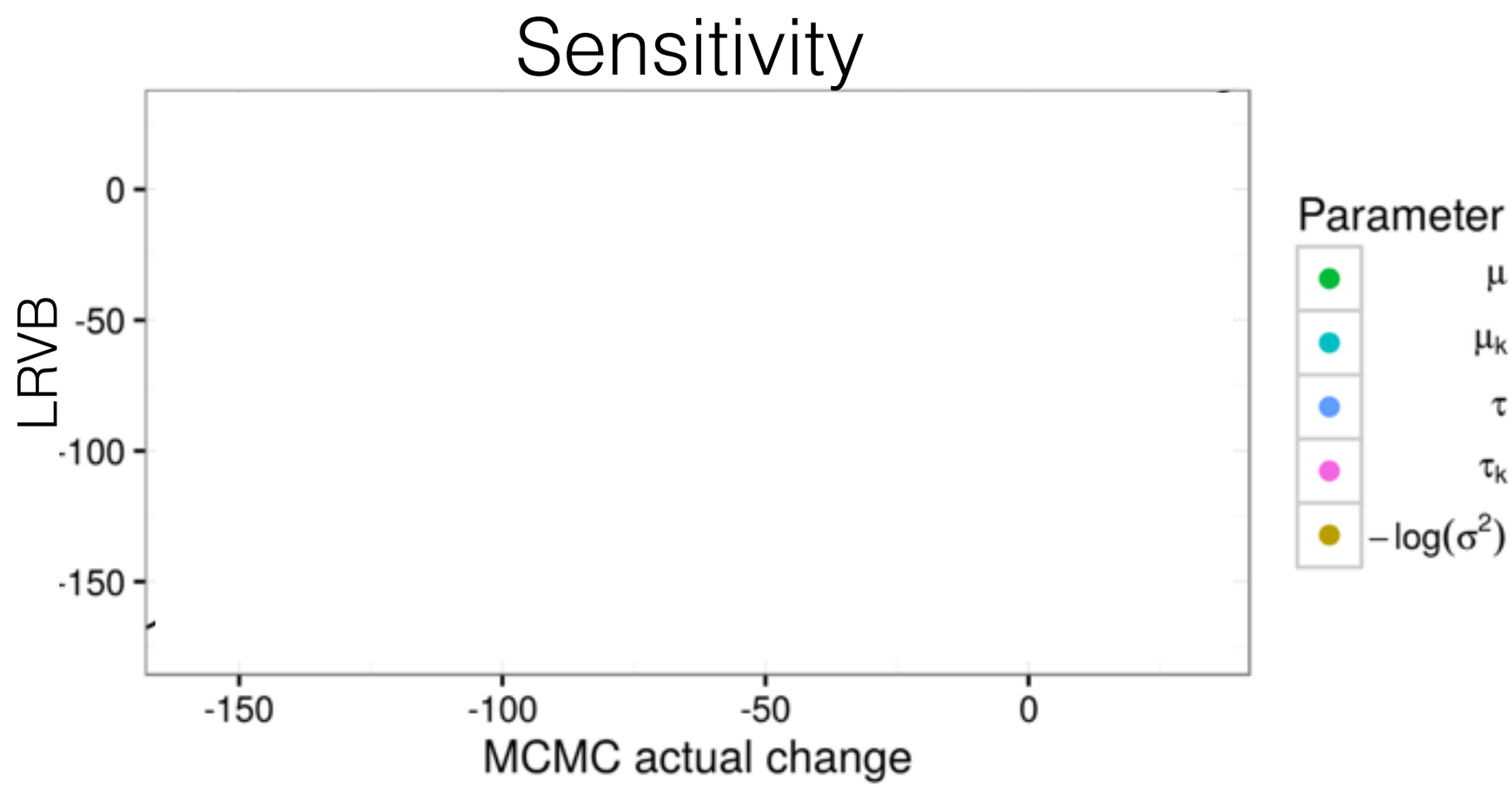

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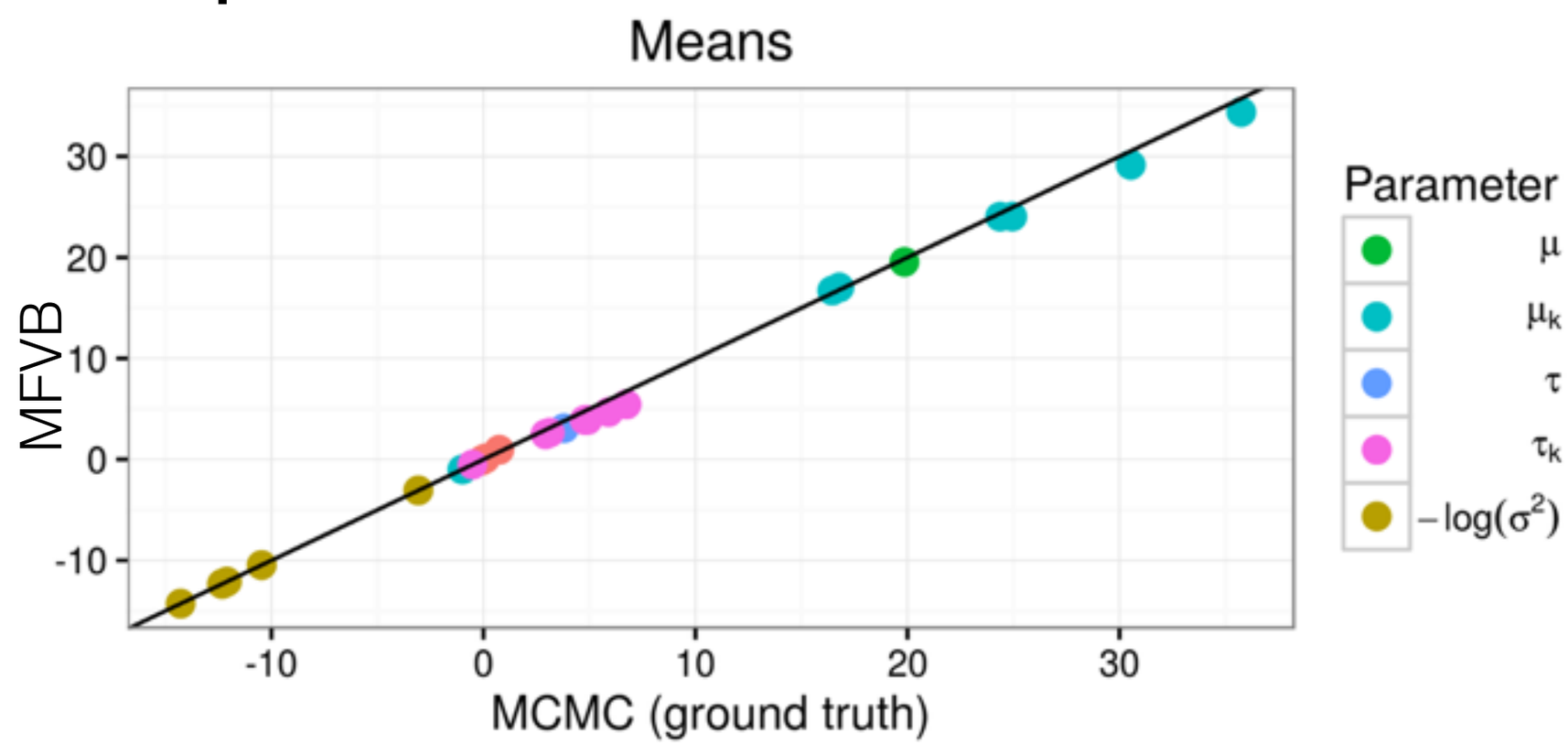

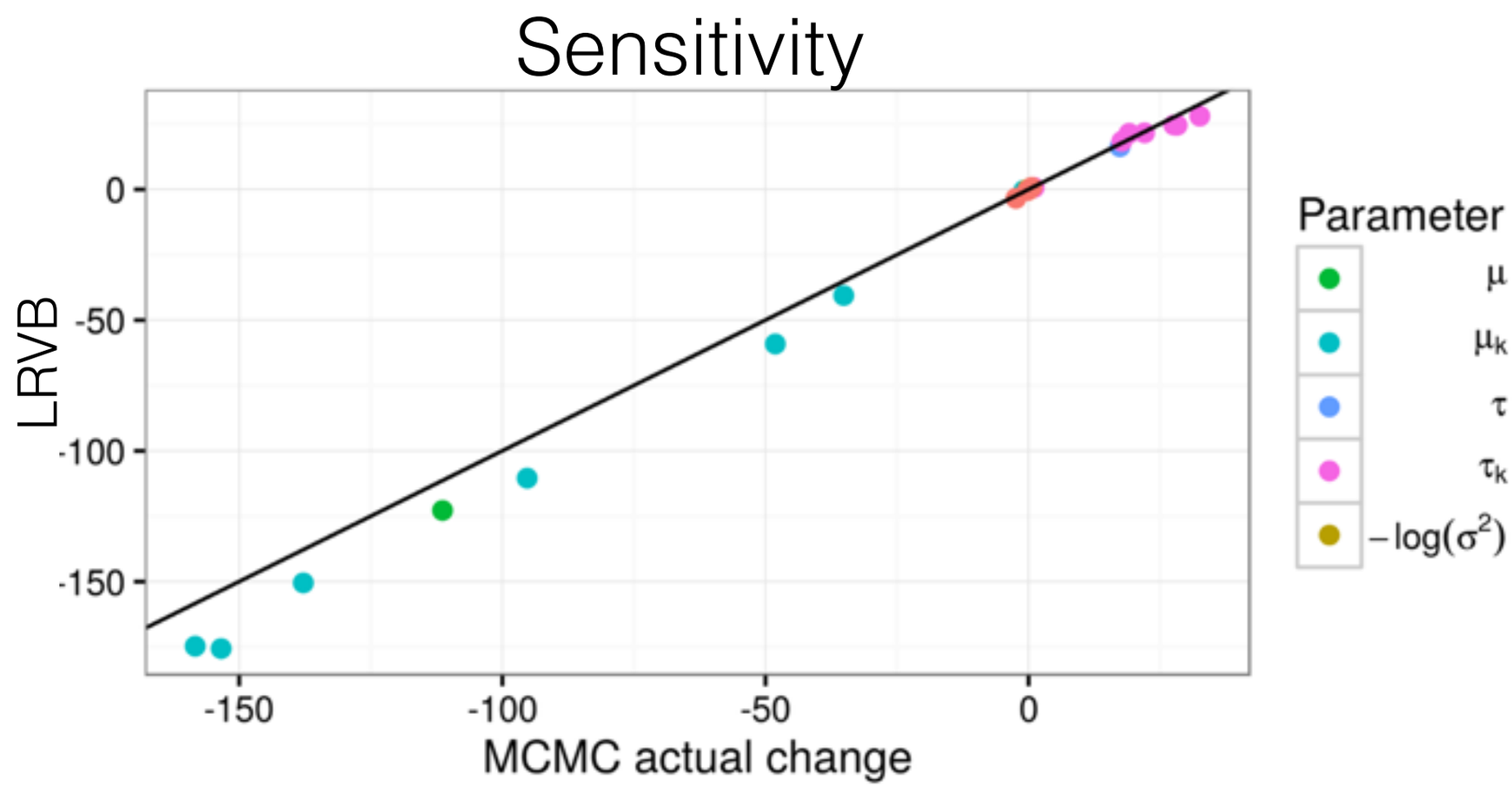

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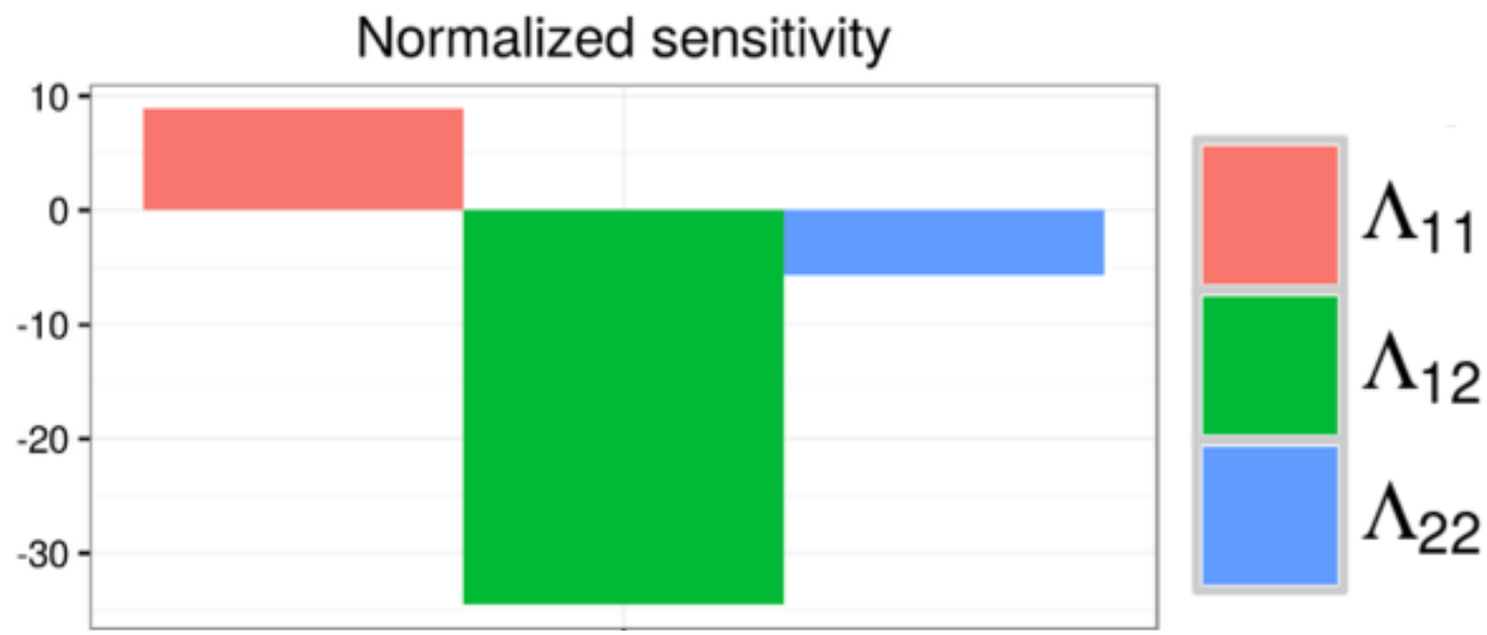

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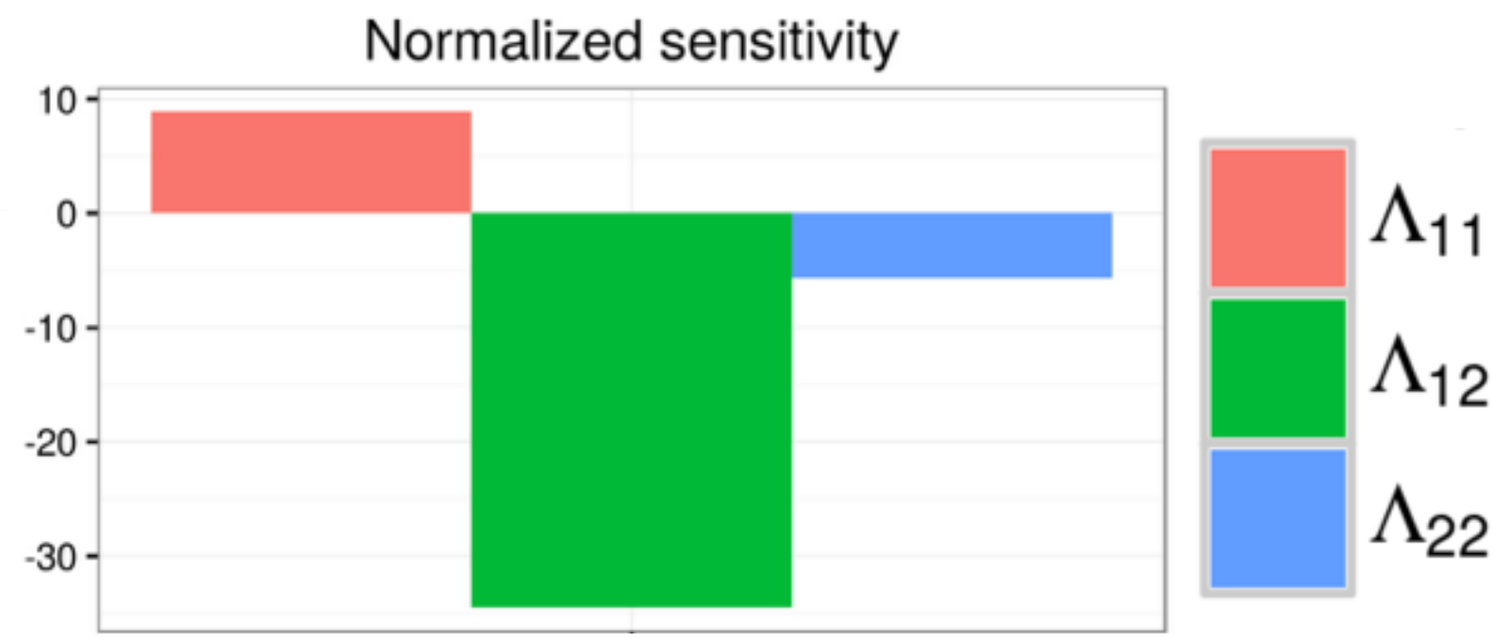

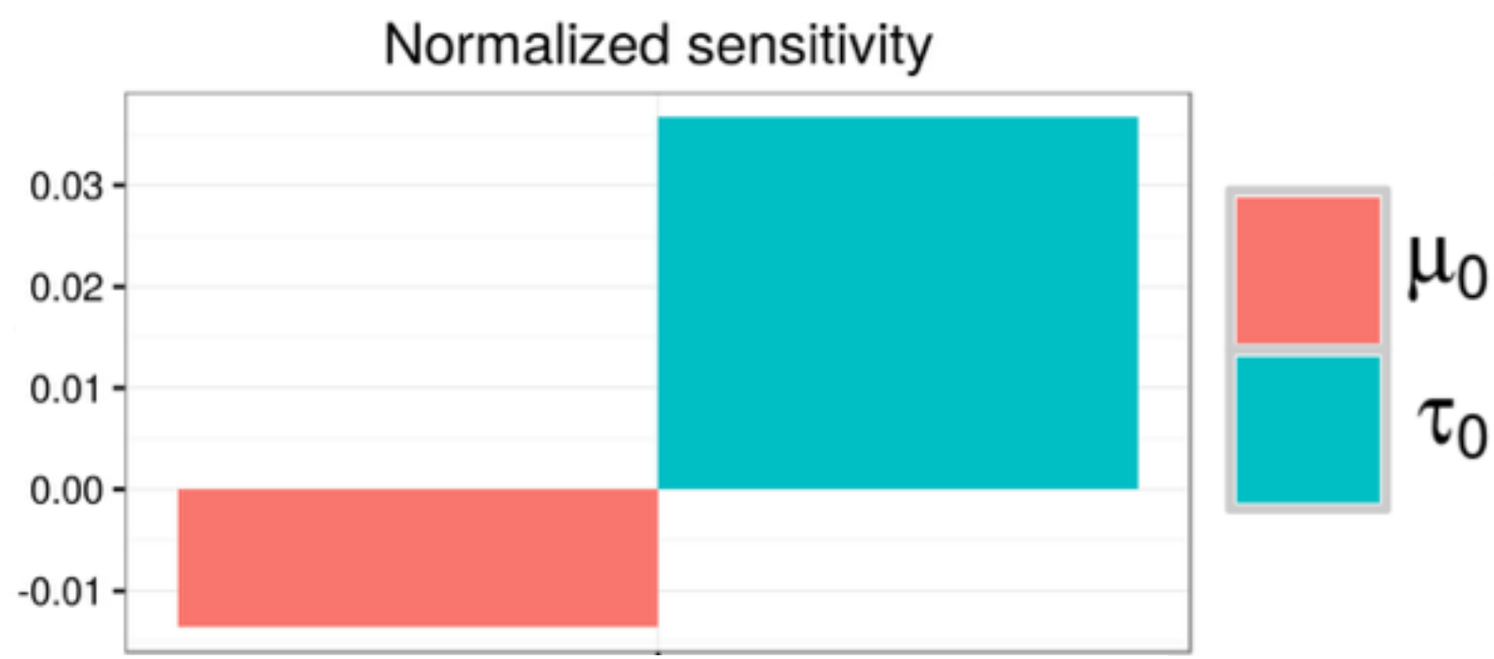

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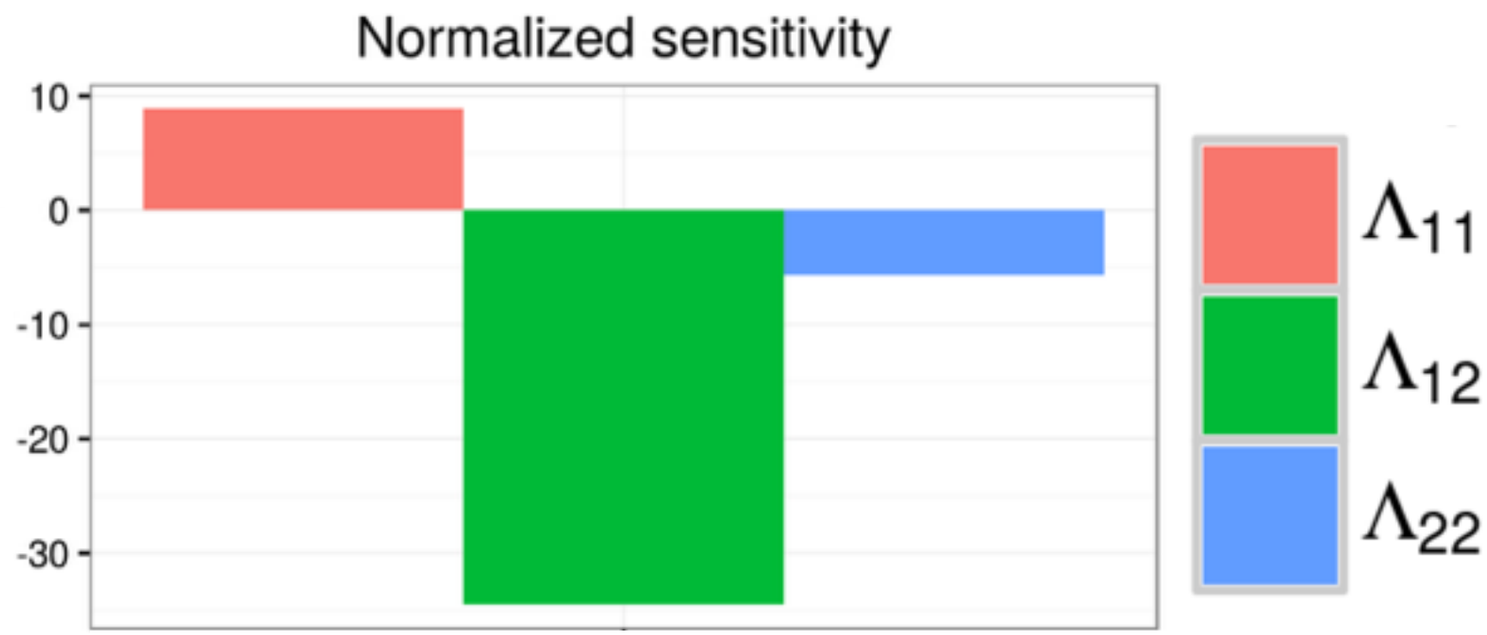

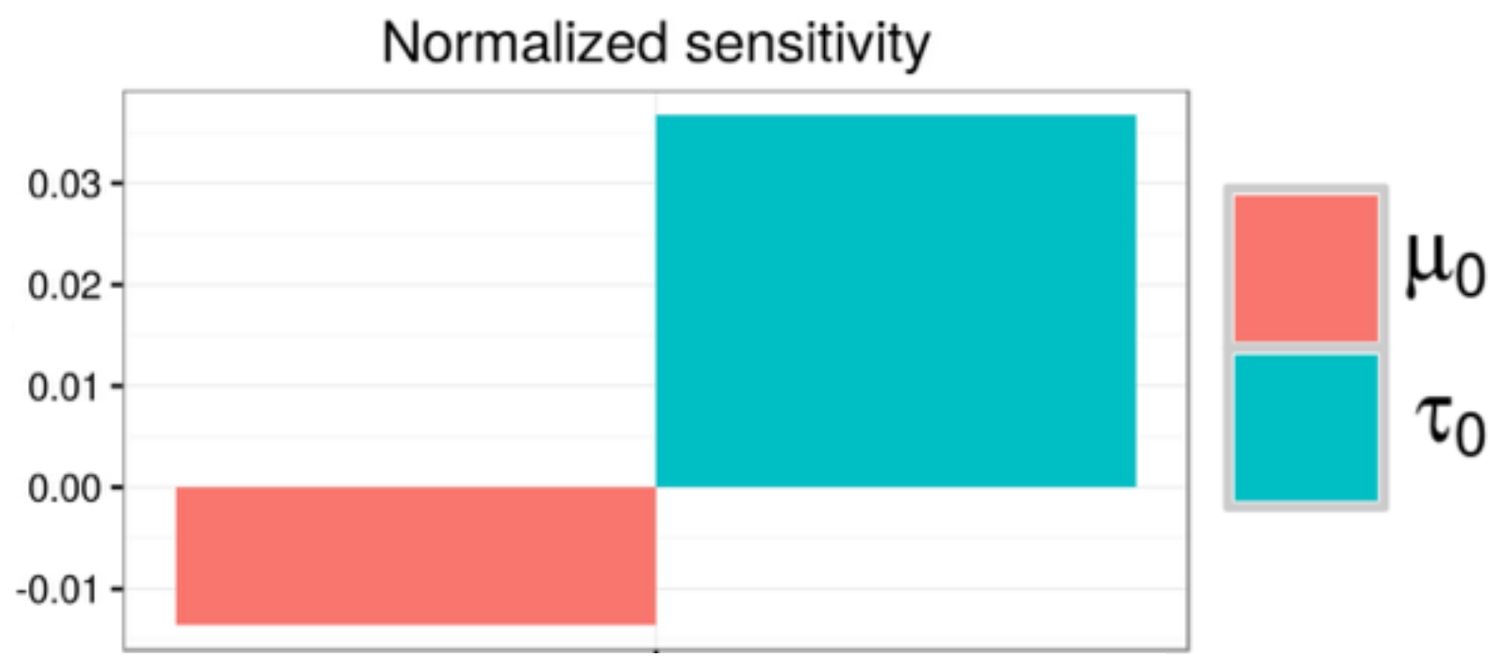

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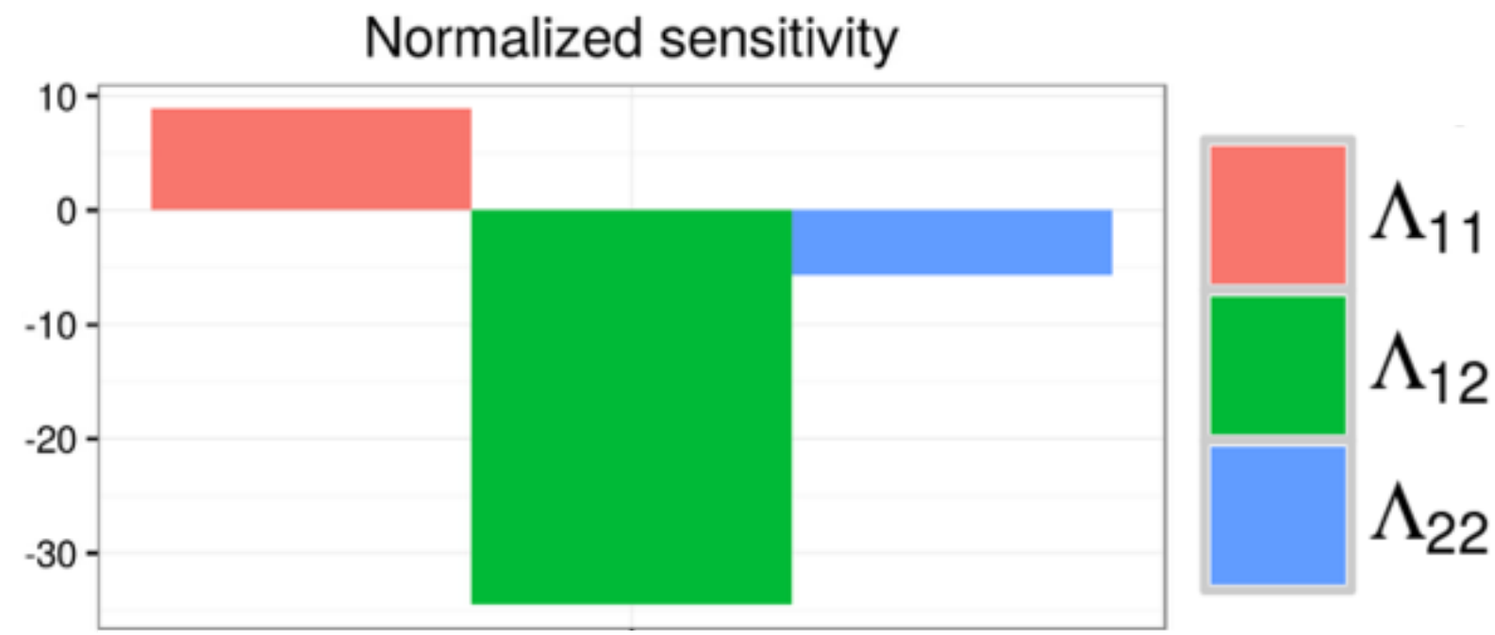

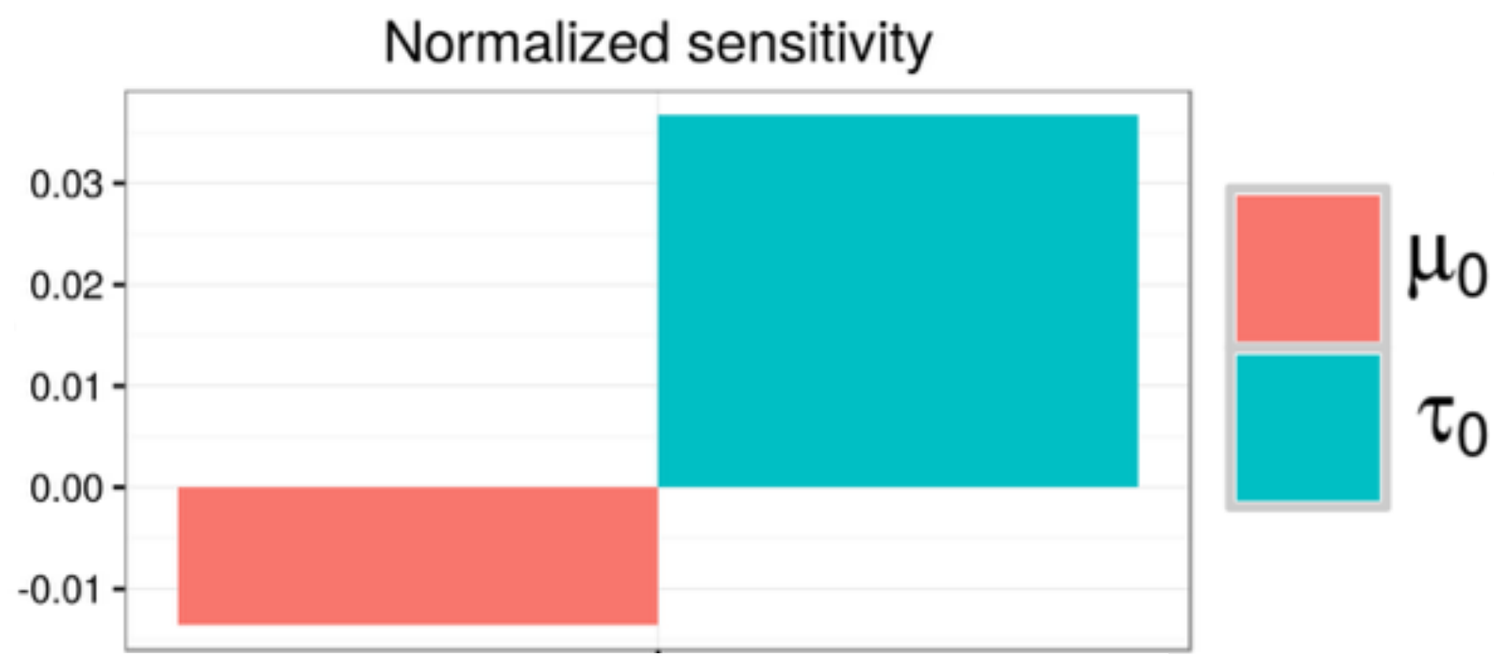

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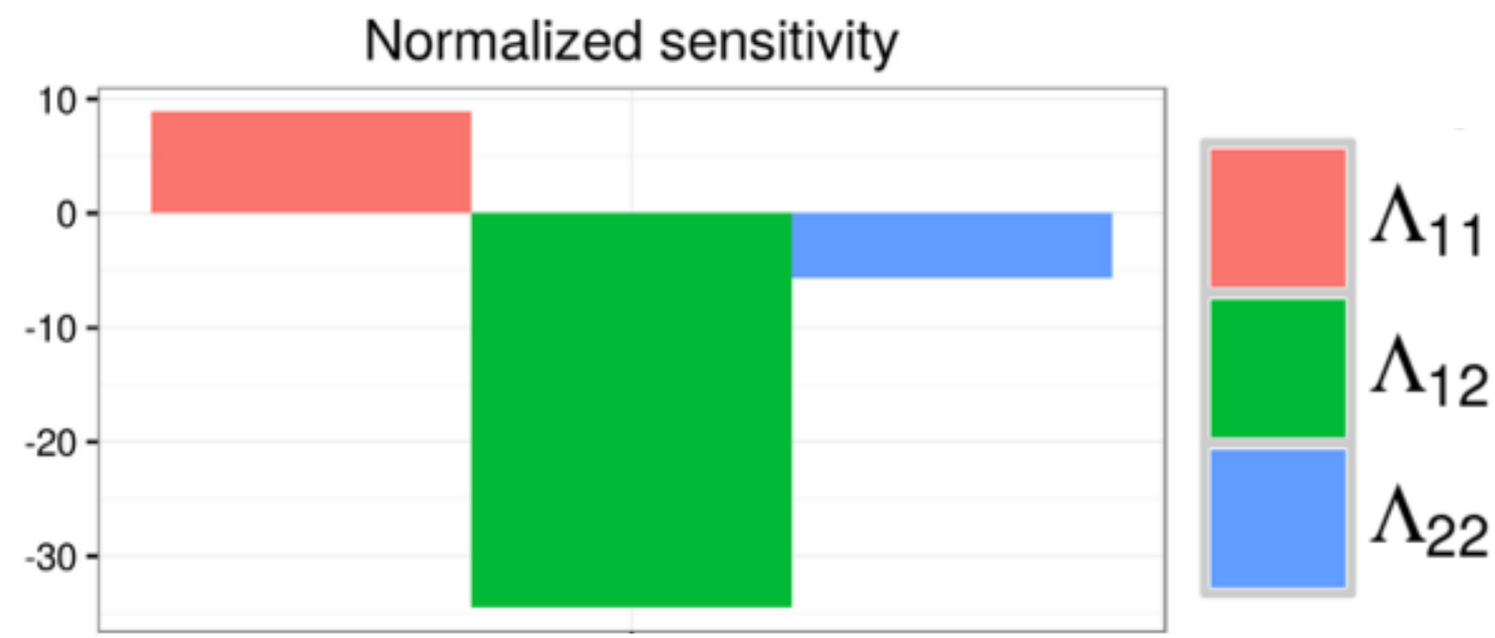
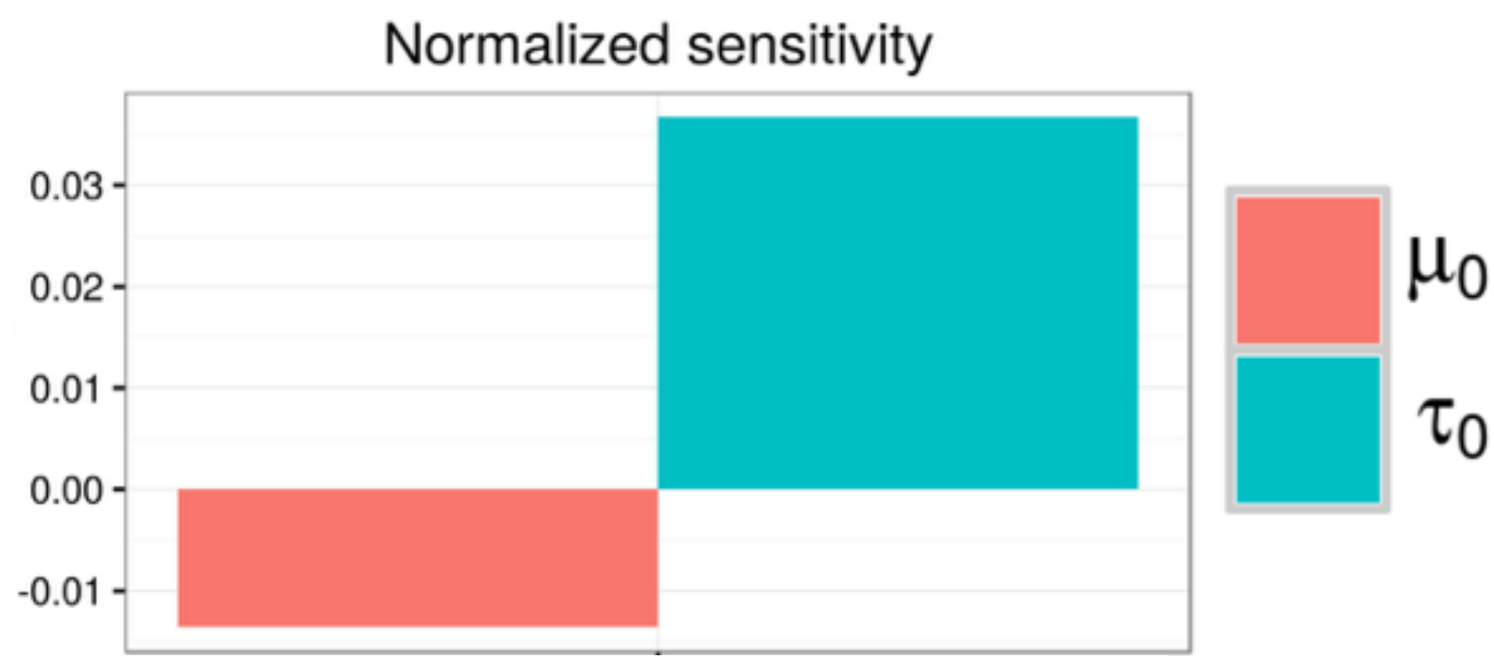
$\Rightarrow$ Mean > 2 std dev





Conclusions

- We provide *linear response variational Bayes*: supplements MFVB for fast & accurate **covariance** estimate
- More from LRVB: fast & accurate **robustness** quantification
- Interested in your data and models:
  - Sensitivity to prior perturbations
  - Sensitivity to likelihood, data perturbations
- Computational statistical trade-offs
  - New data summaries: coresets, approx. sufficient stats
  - Criteo data set: 40 million data points, 3 million features, our runtime: ~20 seconds on 24 cores
  - Theoretical guarantees on finite-sample quality

[Huggins, Campbell, Broderick 2016; Huggins, Adams, Broderick, submitted]

References

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